

# Equilibrium in a DeFi Lending Market

Thomas J. Rivera\*      Fahad Saleh<sup>†</sup>      Quentin Vandeweyer<sup>‡</sup>

September 2026

## Abstract

We provide an economic model of a Decentralized Lending Protocol (DLP) and analyze its novel pricing mechanism. We show that the mechanism admits a unique equilibrium under general conditions, allowing for risk aversion and incomplete information. Additionally, we demonstrate that when borrowers and lenders are fully informed about the state of the credit market, then the DLP can generate approximately efficient interest rates even if the DLP pricing mechanism cannot condition on the state of the credit market. In contrast, we show that if users face uncertainty about the state of the credit market, then any DLP pricing mechanism must generate interest rates that are inefficient. Given these findings, our results highlight that the key inefficiency of the DLP pricing mechanism arises from uncertainty faced by DLP users.

**Keywords:** Decentralized Finance, DeFi, Decentralized Lending Protocols, DLP

---

\*McGill University. Email: [thomas.rivera@mcgill.ca](mailto:thomas.rivera@mcgill.ca)

<sup>†</sup>University of Florida. Email: [fahad.saleh@ufl.edu](mailto:fahad.saleh@ufl.edu)

<sup>‡</sup>London Business School. Email: [qvandeweyer@london.edu](mailto:qvandeweyer@london.edu)

# 1 Introduction

Decentralized finance (DeFi) refers to the provision of traditional financial services via a blockchain, without intermediaries. A particularly prominent example is dis-intermediated lending and borrowing, which is facilitated by a set of smart contracts hereafter referred to as a *Decentralized Lending Protocol (DLP)*. These smart contracts allow investors to freely supply loanable funds from which other agents may borrow. A DLP sets interest rates in a mechanical fashion. Borrowers face an interest rate given by an increasing function of the utilization rate, the ratio of borrowed funds to the funds contributed by lenders. The utilization rate is typically below one, so that a portion of each lender’s funds remains unborrowed. All lenders share the interest payments made by borrowers in proportion to the funds that they contributed (i.e., pro rata). Consequently, an interest rate spread exists whenever the utilization rate is below one: the rate paid by borrowers exceeds the rate received by lenders. This interest rate wedge naturally raises a concern over inefficiency, as it stands in contrast to a competitive market equilibrium in which borrowers and lenders face the same market-clearing interest rate. In this paper, we provide an economic model of a DLP, focusing on the DLP’s interest rate pricing mechanism. We particularly seek to understand whether the DLP interest rate pricing mechanism leads to stable outcomes and whether, and when, such a mechanical market can approximate a competitive market.

With regard to stability, our findings are affirmative. More explicitly, within a dynamic setting, we show that a stationary equilibrium always exists and that this equilibrium is unique. Existence of a stationary equilibrium implies that, although lending and borrowing volumes vary with market conditions, a DLP nonetheless remains functional through adverse market events. More formally, DLP activity adjusts endogenously to market fluctuations and declines in DLP activity revert when market conditions improve. Furthermore, these results hold for a large class of practically relevant interest rate functions and therefore the existence and uniqueness of a stationary equilibrium does not hinge on the specific interest rate function utilized by the DLP.

With regard to efficiency, our findings depend on the information available to users when they make their borrowing and lending decisions. When users know the state of the credit market, we show that there exists an interest rate function under which equilibrium utilization is arbitrarily close to one, irrespective of the state of the credit market. Under such a function, the wedge between the borrowers’ rate and the lenders’ rate is arbitrarily small, and welfare is arbitrarily close to that of a competitive equilibrium. The inefficiency cannot be eliminated entirely, but it can be made arbitrarily small. In contrast, when users face uncertainty about aggregate funding demand and supply, no interest rate function delivers

this approximation. Equilibrium utilization remains bounded away from one with positive probability, leaving some supplied funds unborrowed. Thus, we find that uncertainty *faced by users* implies that the DLP’s interest rate setting mechanism produces inefficient interest rates. Importantly, although DLPs face an oracle problem in obtaining reliable collateral prices from outside the blockchain (see [John et al. 2023](#)), our inefficiency result holds even when collateral values are correctly known. In particular, when users know the state of the credit market, an interest rate setting rule chosen at inception can generate approximately competitive outcomes. In contrast, when users are uncertain about the market state, no interest rate function delivers this approximation.

Formally, we study a dynamic model where a measure of borrowers and lenders arrive to the DLP at the beginning of each period. Borrowers have investment opportunities and can pursue them by borrowing capital from lenders. Loans are initiated at the end of each period and mature at the end of the next period. Each borrower holds a collateral asset, which she posts to the DLP to secure her loan. The collateral earns a random return over the life of the loan. At maturity, each borrower decides whether to repay her loan. If she repays, she recovers her collateral. If she does not, she forfeits her collateral to the DLP. A borrower therefore repays if and only if the value of her collateral covers her repayment obligation. We allow agents to be risk averse and to face uncertainty about the state of the credit market when making their borrowing and lending decisions. Each type of agent determines whether to borrow (lend) at the DLP based on whether the expected utility generated from borrowing (lending) at the platform is higher than the utility from borrowing (lending) at an alternative outside venue. We further assume that credit market conditions follow a stochastic process with some information revealed before decisions are made and then further information revealed after decisions are made. In turn, since some information is revealed after decisions are made, DLP users cannot condition directly on DLP interest rates; rather, DLP users act optimally given the distribution of DLP interest rates that they face.

An important feature of our framework is that we model the interest rate setting procedure of the DLP to match that of DLPs in practice. Specifically, DLPs in practice set a publicly known interest rate function encoded into a smart contract. Once the DLP smart contract is deployed, the interest rate setting rule is fixed for all future periods and therefore must be designed to withstand any future shocks to credit market conditions without conditioning on external information about those conditions. Borrower interest rates are determined by evaluating the DLP’s interest rate function at the observed ratio of borrowed to available loanable funds, hereafter referred to as the *utilization rate*. The lender interest rate is then determined as the rate that ensures that all realized borrower payments, from loan repayments or seized collateral, are allocated pro rata to the lenders who funded those

loans. Notably, our framework not only models the DLP interest rate setting procedure to match the procedures seen in practice but also incorporates the endogeneity of the utilization rate.

Our first main result demonstrates the existence and uniqueness of a stationary DeFi lending equilibrium. This result clarifies that the mechanical interest rate setting procedure of the DLP is not only consistent with equilibrium but that it leads unambiguously to a unique equilibrium. From a practical perspective, our result highlights that DLPs possess some stability. More explicitly, our equilibrium solution demonstrates that DLP economic activity varies with market conditions but that declines in DLP activity revert as market conditions improve. Our unique stationary equilibrium helps explain how DLPs have been able to remain viable despite volatile credit market conditions.

Next, we study the determinants of DeFi lending inefficiencies. We first study a benchmark case whereby users are perfectly informed about the state of the credit market when making their borrowing and lending decisions. Note that while the users are perfectly informed about the state of the credit market, the credit market still evolves stochastically in each period and therefore any DLP smart contract that sets its interest rate function in a particular period will be effectively uninformed about, and therefore unable to condition on, the state of the credit market in each subsequent period. Despite the DLP smart contract being unable to condition on the state of the credit market, we demonstrate that a well designed interest rate function can generate equilibrium DLP interest rates that are arbitrarily close to market-clearing interest rates in each period, irrespective of the state of the credit market. Therefore, even if it were possible to perfectly inform the smart contract of the state of the credit market in each period, the efficiency gains would be negligible at a well designed DLP.

The key to overcoming the DLP’s lack of information about the state of the credit market when users are perfectly informed is to design an interest rate function that leverages the user’s knowledge of the state to provide them with the correct borrowing and lending incentives. Such interest rate functions have a simple design. In particular, market clearing requires equilibrium utilization rates to be close to unity in order to minimize excess supply, and the DLP design must ensure that interest rates never exceed unity as this would imply more borrowing than available funds. Therefore, an interest rate function that sets prohibitively low lending rates when utilization is slightly below unity and prohibitively high borrowing rates when utilization is equal to unity will ensure that the equilibrium utilization rates are always close but not equal to unity. The reason that this design guarantees such high utilization rates is that a user knows that, for any state of the credit market, if the utilization rate were to realize as far below unity, then borrowing and lending rates would

be especially low. Yet, in that case, borrowers would have a profitable deviation to borrow more at those low rates and lenders would have a profitable deviation to lend less to the DLP by diverting capital to higher return alternative investment opportunities. Note that both increased borrowing and decreased lending increase the utilization rate, which is why any interest rate function satisfying the aforementioned properties can ensure equilibrium utilization rates near unity. Crucially, our results imply that equilibrium utilization rates can be targeted to be arbitrarily close to unity, e.g., above 90%, by setting prohibitively low interest rates when utilization is below 90% and prohibitively high interest rates when utilization is 100%. Nonetheless, as we discuss subsequently in detail, such interest rate functions will not produce efficient interest rates if DLP users face uncertainty regarding the state of the credit market when making their borrowing and lending decisions.

We now elaborate on the key inefficiency of the DLP interest rate setting procedure. In particular, when users face uncertainty about the state of the credit market, we demonstrate that the DLP will feature equilibrium utilization that is uniformly bounded away from unity with a strictly positive probability where the bound is uniform over all possible DLP interest rate functions. Therefore, a DLP will always feature excess supply in equilibrium. This excess supply represents a key inefficiency of DeFi lending as this capital could be reallocated to productive use rather than sitting idle in the platform.

The cause of the aforementioned DLP inefficiencies stems from the mechanical nature of the DLP interest rate setting procedure which ensures that noise in the state of the credit market will lead to noise in the DLP borrowing and lending rates. As a consequence, risk-averse borrowers will be discouraged from borrowing and lending at the DeFi lending platform if noise in the equilibrium interest rates can produce unfavorable terms. As discussed, utilization in excess of unity is not feasible at a DLP as this would require that more funds are borrowed than are lent to the platform. For this reason, DLPs in practice set prohibitively high interest rates when utilization approaches unity (e.g., rates in excess of 50%). This produces a key trade-off for the DLP. Namely, minimizing the DLP inefficiencies requires minimizing realized excess supply at the DLP which requires the DLP to generate equilibrium utilization rates that are sufficiently high. Yet, if a DLP aims to generate sufficiently high utilization rates in each period (e.g., by using interest rate functions with the aforementioned properties), then shocks to the credit market will lead to shocks to the utilization rate, causing the utilization to approach unity with positive probability. Therefore, noise in the credit market conditions would imply that there is a positive probability that borrowers face prohibitively large borrowing rates when the DLP aims to generate sufficiently high utilization rates. Yet, borrowers internalize these bad outcomes when making their borrowing decisions which leads to less ex-ante borrowing demand and therefore lower

utilization, undermining the aim of the DLP. Thus, a DLP that generates positive borrowing demand cannot eliminate excess supply for all realizations of the state of the credit market. As discussed above, this excess supply is inefficient in the sense that it could be reallocated to alternative activities in a way that generates strictly higher welfare. Finally, we note that the provision of offchain data to the DLP does not, by itself, resolve this inefficiency, as it is precisely information that is unknown to the users that prevents the DLP from setting optimal interest rates. In summary, inefficiencies at a well designed DLP arise from user uncertainty about credit market conditions. We state all our economic findings in the manuscript and defer all proofs to the online appendix.

**Literature** Our paper contributes to the literature that studies the economics of blockchain technology. While much of the early work in that literature examines economic security (see, e.g., [Biais et al. 2019](#), [Easley et al. 2019](#), [Rosu and Saleh 2021](#), [Saleh 2021](#), and [John et al. 2025](#)), we abstract from such concerns and focus instead upon economic implications that arise from a secure blockchain. We specifically examine a secure blockchain with smart contract functionality (e.g., Ethereum) and study welfare implications arising from a prominent application, borrowing and lending facilitated by a DLP. The economics literature that studies blockchains with smart contract functionality is young and growing (see [John et al. 2023](#) and [John and Saleh 2025](#)) - topics of study within this literature include tokenomics (see, e.g., [Tsoukalas and Falk 2020](#), [Cong et al. 2021](#) and [Mayer 2022](#)), stablecoins (see, e.g., [Li and Mayer 2022](#) and [d’Avernas et al. 2026](#)), and decentralized exchanges (see, e.g., [Park 2023](#), [Cao et al. 2025](#), [Capponi and Jia 2025](#), [Lehar and Parlour 2025](#), [Hasbrouck et al. 2026a](#) and [Hasbrouck et al. 2026b](#)). We add to the smart contracts literature by providing a formal economic model of a DLP with a particular focus on the DLP’s interest rate setting mechanism. Other notable papers that also examine DLPs include [Aramonte et al. \(2022\)](#), [Carré and Gabriel \(2022\)](#), [Chiu et al. \(2022\)](#), [Mueller \(2022\)](#), [Lehar and Parlour \(2023\)](#), [Wang \(2024\)](#), and [Dong et al. \(2025\)](#). [Aramonte et al. \(2022\)](#) highlight that DLPs serve largely as a vehicle for leveraged cryptoasset trading. [Chiu et al. \(2022\)](#) theoretically model a DLP while focusing on asymmetric information between borrowers and lenders. [Mueller \(2022\)](#), [Lehar and Parlour \(2023\)](#), [Wang \(2024\)](#), and [Dong et al. \(2025\)](#) provide empirical insights regarding DLPs. Our work differs from prior DLP papers in that we focus upon the interest rate setting mechanism and the associated implications for efficiency. Finally, our results relate to the oracle problem, i.e. the ability to integrate data to blockchains from external environments as studied by [Cong et al. \(2026\)](#).

## 2 Institutional Detail Regarding Decentralized Lending Protocols

A DeFi lending market is characterized by a Decentralized Lending Protocol (DLP), which serves a similar role to that of a financial intermediary within a traditional lending market. The DLP is a set of smart contracts that encodes transparent rules for setting interest rates. The DLP rules are typically defined by a single pre-determined interest rate function which sets the borrower interest rate directly and the lender interest rate indirectly. Specifically, the borrower interest rate is given by a particular point on the DLP interest rate function, whereas the lender interest rate is the implied rate from passing borrower payments onto lenders.

A DLP sets the borrower interest rate by inputting onchain information regarding lending supply and borrowing demand into the previously referenced interest rate function. More precisely, the DLP sets its borrowing interest rate as a function of the *utilization rate*, defined as

$$\text{Utilization Rate} = \frac{\text{Observed Borrowing}}{\text{Observed Lending}}. \quad (2.1)$$

In turn, the borrower's interest rate is determined as

$$\text{Borrower IR} = \rho(\text{Utilization Rate}), \quad (2.2)$$

where  $\rho$  denotes the exogenous interest rate function that defines the DLP. The interest rate function,  $\rho$ , is specified as an increasing function of the utilization rate so that borrower interest rates increase with observed borrowing demand and decrease with observed lending supply. Then, since the DLP passes borrower payments onto lenders, the DLP lender interest rate must satisfy the following equation:<sup>1</sup>

$$\underbrace{\text{Observed Lending} \cdot \text{Lender IR}}_{\text{Lender Receipts}} = \underbrace{\text{Observed Borrowing} \cdot \text{Borrower IR}}_{\text{Borrower Payments}} \quad (2.3)$$

Equations (2.2) and (2.3) then yield the mechanical rule for determining the DLP lender interest rate:

$$\text{Lender IR} = \text{Utilization Rate} \cdot \rho(\text{Utilization Rate}) \quad (2.4)$$

Note that, given a utilization rate, Equation (2.2) and (2.4) correspond to mechanical rules for setting interest rates. However, the utilization rate is an endogenous quantity and

---

<sup>1</sup>In practice, a DLP generally passes a fixed proportion of payments onto the lender. That proportion is generally close to 100%, so we abstract from this detail and assume a 100% pass-through.

its value depends upon both economic conditions and the mechanical interest rate setting rule. In particular, investors internalize the mechanical interest rate setting rules and determine their lending and borrowing on that basis. In turn, lending and borrowing decisions determine the utilization rate which determines DLP interest rates. A comprehensive analysis of a DLP thus must necessarily incorporate this endogeneity whereby the utilization rate is itself determined by the mechanical interest setting rules. Our framework specifically incorporates that endogeneity.

### 3 Model

We examine a discrete time infinite horizon model where time is indexed by  $t \in \mathbb{N}$  and where there exists a single DLP with borrower interest rate function  $\rho : [0, 1] \mapsto \mathbb{R}_+$ . At the beginning of each period, a random measure of lenders and a random measure of borrowers arrive to the lending market. Lenders possess a unit of capital and heterogeneous investment opportunities so that each lender may either lend to the DLP or invest in an alternative investment opportunity. Borrowers possess no liquid capital but have access to an investment opportunity. Borrowers may borrow from the DLP or at an alternative borrowing rate which is heterogeneous across borrowers. Each borrower who borrows from the DLP posts collateral to secure her loan. The measures of borrowers and lenders that arrive in each period have time series dependence, and the measures from prior periods are public information. In contrast, the innovations to these measures are not fully known until the end of the period. In turn, borrowers and lenders choose whether to utilize the DLP based on the distribution of rates and returns that they face given their most recent information. At the end of period  $t$ , the measures of borrowers and lenders are fully revealed, loans are initiated, and utilization determines their borrowing rate. Each loan finances an investment over one period and matures at the end of period  $t + 1$ . At that maturity, the investment and collateral returns are realized, and the borrower either repays her loan with interest or defaults, in which case the DLP seizes her collateral. The DLP allocates borrower repayments or seized collateral pro rata to the lenders who funded those loans, together with their unborrowed capital. Alternative borrowing and investment opportunities have the same one-period horizon. The next cohort of borrowers and lenders arrives at the beginning of period  $t + 1$ , and its loans are initiated at the end of that period using the new lenders' funds.

### 3.1 Lenders and Borrowers

At the beginning of each period, a measure  $\mu_t$  of borrowers and a measure  $\lambda_t$  of lenders arrive where both  $\{\mu_t\}_{t=1}^\infty$  and  $\{\lambda_t\}_{t=1}^\infty$  constitute strictly positive sequences. For simplicity, we assume that the log of each process follows an AR(1) as follows:

$$\mu_t = \mu_{t-1}^\phi \cdot \varepsilon_t^\mu, \quad \lambda_t = \lambda_{t-1}^\phi \cdot \varepsilon_t^\lambda \quad (3.1)$$

where  $\phi \in (0, 1)$  while both  $\{\varepsilon_t^\mu\}_{t=1}^\infty$  and  $\{\varepsilon_t^\lambda\}_{t=1}^\infty$  are strictly positive i.i.d sequences. Hereafter, we denote by  $F_\mu$  and  $F_\lambda$  the respective stationary distributions for  $\{\mu_t\}_{t=0}^\infty$  and  $\{\lambda_t\}_{t=0}^\infty$ .<sup>2</sup>

We decompose the incremental information revealed in each period as follows:

$$\varepsilon_t^\mu = \varepsilon_t^{\mu,-} \cdot \varepsilon_t^{\mu,+}, \quad \varepsilon_t^\lambda = \varepsilon_t^{\lambda,-} \cdot \varepsilon_t^{\lambda,+} \quad (3.2)$$

where  $\varepsilon_t^{\mu,-}$  and  $\varepsilon_t^{\lambda,-}$  denote information revealed at the beginning of period  $t$  (before borrowing and lending decisions are made) about the measure of borrowers and lenders arriving respectively in period  $t$ . Additionally,  $\varepsilon_t^{\mu,+}$  and  $\varepsilon_t^{\lambda,+}$  denote information revealed at the end of period  $t$  about the measure of borrowers and lenders arriving in period  $t$  (after borrowing and lending decisions are made). Note that we include both beginning and end of period  $t$  shocks to capture important features of DeFi lending such as the DLP's inability to incorporate known shocks to the state of the credit market into its interest rate procedure (due to beginning of period shocks) as well as the users' inability to perfectly anticipate the state of the credit market when making their decisions (due to end of period shocks). We assume that the innovation vectors  $\{(\varepsilon_t^{\mu,-}, \varepsilon_t^{\lambda,-}, \varepsilon_t^{\mu,+}, \varepsilon_t^{\lambda,+})\}_{t=1}^\infty$  are i.i.d across periods, that each of the four innovations has a finite support over the positive real line and that, within each period, the end of period innovations  $(\varepsilon_t^{\mu,+}, \varepsilon_t^{\lambda,+})$  are independent of the beginning of period innovations  $(\varepsilon_t^{\mu,-}, \varepsilon_t^{\lambda,-})$ . Formally, we denote the probability mass function of  $\varepsilon_t^{\mu,-}$  by  $\pi_-^\mu$  and the probability mass function of  $\varepsilon_t^{\mu,+}$  by  $\pi_+^\mu$ , and we denote their supports as  $S_-^\mu \subseteq \mathbb{R}_+$  and  $S_+^\mu \subseteq \mathbb{R}_+$  respectively. Similarly, we denote the probability mass function of  $\varepsilon_t^{\lambda,-}$  by  $\pi_-^\lambda$  and the probability mass function of  $\varepsilon_t^{\lambda,+}$  by  $\pi_+^\lambda$ , and we denote their supports as  $S_-^\lambda \subseteq \mathbb{R}_+$  and  $S_+^\lambda \subseteq \mathbb{R}_+$  respectively.

As we shall see,  $\mu_t$  and  $\lambda_t$  affect the equilibrium solutions only through their ratio (i.e.,  $\frac{\mu_t}{\lambda_t}$ ) so that it is convenient to define an additional random sequence,  $\{\theta_t\}_{t=1}^\infty$ , as follows:

$$\theta_t := \frac{\mu_t}{\lambda_t} \quad (3.3)$$

---

<sup>2</sup>Note that these distributions are given explicitly as the distributions for random variables  $e^{X_\mu}$  and  $e^{X_\lambda}$  respectively where  $X_\mu \stackrel{d}{=} \sum_{k=0}^\infty \phi^k \log \varepsilon_k^\mu$  and  $X_\lambda \stackrel{d}{=} \sum_{k=0}^\infty \phi^k \log \varepsilon_k^\lambda$ .

where  $\theta_t$  captures the state of the credit market from the borrower's perspective with high values of  $\theta_t$  corresponding to high credit market tightness — relatively many borrowers as compared to lenders (i.e., high  $\mu_t$  relative to  $\lambda_t$ ) — and low values of  $\theta_t$  corresponding to low credit market tightness — relatively few borrowers as compared to lenders (i.e., low  $\mu_t$  relative to  $\lambda_t$ ). Note that Equation (3.1) directly implies that the log of  $\{\theta_t\}_{t=1}^\infty$  also follows an AR(1) process as follows:

$$\theta_t = \theta_{t-1}^\phi \cdot \varepsilon_t \quad (3.4)$$

where  $\varepsilon_t = \frac{\varepsilon_t^\mu}{\varepsilon_t^\lambda}$  represents new information in period  $t$ .  $\varepsilon_t$  can then be decomposed as follows:

$$\varepsilon_t = \varepsilon_t^- \cdot \varepsilon_t^+ \quad (3.5)$$

where  $\varepsilon_t^- = \frac{\varepsilon_t^{\mu,-}}{\varepsilon_t^{\lambda,-}} > 0$  and  $\varepsilon_t^+ = \frac{\varepsilon_t^{\mu,+}}{\varepsilon_t^{\lambda,+}} > 0$ . Hereafter, we reference the probability mass function of  $\varepsilon_t^+$  by  $\pi_+$  and the probability mass function of  $\varepsilon_t^-$  by  $\pi_-$ . Moreover, we denote their supports by  $S_+$  and  $S_-$  respectively, with  $\varepsilon_{\max}^+$  denoting the largest element of  $S_+$ .

As a final notational point, recall that borrowers and lenders choose their venues at the beginning of period  $t$  (i.e., with knowledge of  $\varepsilon_t^-$  but not necessarily  $\varepsilon_t^+$ ). In turn, it is useful to specify the most recent information available at that venue choice by  $\tilde{\theta}_t$  as follows:

$$\tilde{\theta}_t = \theta_{t-1}^\phi \cdot \varepsilon_t^- \quad (3.6)$$

### 3.1.1 Lenders

We uniquely identify each lender by the ordered pair  $(j, t) \in [0, \lambda_t] \times \mathbb{N}$  where  $t$  denotes the period in which the lender arrives and  $j$  denotes the index of each lender among the lenders that arrive in period  $t$ . We assume that Lender  $(j, t)$  possesses a unit of capital and has access to an alternative investment opportunity with net return  $r_{l,(j,t)} \sim F_l$  where  $F_l$  denotes a continuously differentiable cumulative distribution function that is strictly increasing on its support  $[0, \infty)$  and satisfies  $F_l(0) = 0$ , i.e., all lenders require a strictly positive return. Lender  $(j, t)$  may invest her capital in the DLP or in her alternative investment opportunity. Denoting by  $l_t \geq 0$  the DLP lending rate, her utility,  $\mathcal{V}_{l,(j,t)}(l_t)$ , is therefore given as follows:

$$\mathcal{V}_{l,(j,t)}(l_t) = \max\{\mathbb{E}[v_l(1 + l_t) \mid \tilde{\theta}_t], v_l(1 + r_{l,(j,t)})\} \quad (3.7)$$

where  $v_l : \mathbb{R} \mapsto \mathbb{R}$  denotes the period utility function of each lender and the expectation of the DLP payoff is taken conditional on  $\tilde{\theta}_t$ , reflecting the information available to Lender  $(j, t)$  at the beginning of period  $t$ . We assume that  $v_l$  is twice continuously differentiable and strictly increasing. We also assume that aggregate lender utility from the outside option is

integrable (i.e.,  $\int_0^\infty |v_l(1+x)| dF_l(x) < \infty$ ).

Note that lenders optimally lend to the DLP if and only if their alternative investment payoff is less than or equal to the expected payoff from lending to the DLP (i.e.,  $\mathbb{E}[v_l(1+l_t)|\tilde{\theta}_t] \geq v_l(1+r_{l,(j,t)})$ ). More formally, there exists a cut-off alternative investment lending rate below which all lenders lend to the DLP. This rate,  $r_{l,t}$ , is conditional on the most recent information,  $\tilde{\theta}_t$ , and is given as follows:

$$r_{l,t} = r_{l,t}(\tilde{\theta}_t) = v_l^{-1}(\mathbb{E}[v_l(1+l_t) | \tilde{\theta}_t]) - 1 \quad (3.8)$$

and thus given that all lenders  $(j, t)$  with outside lending rates  $r_{l,(j,t)} \leq r_{l,t}$  optimally lend to the DLP, the period  $t$  lender supply for the DLP,  $\mathcal{S}_t$ , is given explicitly as follows:

$$\mathcal{S}_t = \lambda_t \cdot F_l(r_{l,t}(\tilde{\theta}_t)) \quad (3.9)$$

### 3.1.2 Borrowers

We uniquely identify each borrower by the ordered pair  $(i, t) \in [0, \mu_t] \times \mathbb{N}$  where  $t$  denotes the period in which the borrower arrives and  $i$  denotes the index of the borrower among the borrowers within period  $t$ . We assume that Borrower  $(i, t)$  holds one unit of a collateral asset with market value  $\chi > 0$  at loan origination, has access to an investment opportunity with net return  $y > 0$  and can borrow capital from an outside venue at rate  $r_{b,(i,t)} \sim F_b$  where  $F_b$  denotes a continuously differentiable cumulative distribution function that is strictly increasing on its support  $[\underline{r}_b, \bar{r}_b] \subseteq [0, y)$  with  $\bar{r}_b > 0$ . Each borrower who borrows from the DLP posts her collateral asset to secure her loan. The collateral earns a random net return  $\psi_{t,t+1} \sim F_\psi$  over the life of the loan, so that its value at maturity is  $\chi(1 + \psi_{t,t+1})$ . Our framework thus assumes exogenous collateral returns, reflecting the case where the DLP restricts collateral to liquid assets.<sup>3</sup> We further assume that  $F_\psi$  is continuously differentiable and strictly increasing on its support  $[\underline{\psi}, \bar{\psi}]$  where  $-1 < \underline{\psi} \leq \bar{\psi} < \infty$ , and that  $\{\psi_{t,t+1}\}_{t=1}^\infty$  is i.i.d across periods and independent of all else. Finally, we assume that the collateral is of high quality in the sense that its return distribution,  $F_\psi$ , belongs to a set of probability

---

<sup>3</sup>Standard asset pricing theory implies that the prices of liquid assets reflect only information and preferences (Ross 1978, Harrison and Kreps 1979). In particular, arbitrageurs rationally absorb order imbalances that arise from trades unrelated to information. Liquidations of lending protocol collateral are examples of such trades. Absent limits to arbitrage, their price effects are transitory (Shleifer and Vishny 1997). Crucially, the literature on fire sales establishes that frictions, such as scarce or slow-moving arbitrage capital and concentrated exposures, are necessary for liquidations to cause persistent price effects (see, e.g., Shleifer and Vishny 1992, Coval and Stafford 2007, Duffie 2010, Geanakoplos 2010). These frictions do not arise for liquid assets, and thus our assumption of exogenous collateral returns reflects prudent restrictions by lending protocols on the assets accepted as collateral. Our results should not be read as speaking to thin or manipulable collateral.

distributions,  $\mathcal{C}$ , which is closed under first-order stochastic dominance.<sup>4</sup> This assumption reflects that DLPs impose restrictions on the assets that can serve as collateral precisely to ensure that strategic default is rarely optimal.

We assume that Borrower  $(i, t)$  may invest one unit of capital in the investment opportunity. If she borrows that unit from the DLP at the borrowing rate  $b_t \geq 0$ , then at maturity she either repays her loan or defaults, in which case the DLP seizes her collateral. In turn, denoting by  $v_b : \mathbb{R} \mapsto \mathbb{R}$  the period utility function for borrowers, her payoff from borrowing via the DLP,  $\mathcal{V}_b^{DLP}(b_t)$ , is given as follows:

$$\mathcal{V}_b^{DLP}(b_t) = \max \left\{ \underbrace{v_b(\chi(1 + \psi_{t,t+1}) + y - b_t)}_{\text{repay}}, \underbrace{v_b(1 + y)}_{\text{default}} \right\} \quad (3.10)$$

Instead, if she borrows from her outside venue at rate  $r_{b,(i,t)}$ , then she repays with certainty<sup>5</sup> and her payoff,  $\mathcal{V}_{b,(i,t)}^{oo}$ , is given as follows:

$$\mathcal{V}_{b,(i,t)}^{oo} = v_b(\chi(1 + \psi_{t,t+1}) + y - r_{b,(i,t)}) \quad (3.11)$$

Thus, Borrower  $(i, t)$ 's utility,  $\mathcal{V}_{b,(i,t)}(b_t)$ , when facing a borrowing rate of  $b_t$  is given as follows:

$$\mathcal{V}_{b,(i,t)}(b_t) = \max \left\{ \mathbb{E}[\mathcal{V}_b^{DLP}(b_t) \mid \tilde{\theta}_t], \mathbb{E}[\mathcal{V}_{b,(i,t)}^{oo}] \right\} \quad (3.12)$$

where the first expectation is conditional on  $\tilde{\theta}_t$ , reflecting the information available when the borrower chooses her borrowing venue. The second expectation runs over the collateral return alone, which is independent of  $\tilde{\theta}_t$ . We assume that  $v_b$  is twice continuously differentiable, strictly increasing and weakly concave.

Note that borrowers optimally borrow from the DLP if and only if the expected utility from borrowing via the DLP exceeds that from borrowing through the heterogeneous alternative borrowing opportunity. In turn, there exists a cut-off outside option borrower rate above which all borrowers borrow from the DLP. This rate,  $r_{b,t} = r_{b,t}(\tilde{\theta}_t)$ , is conditional on the most recent information,  $\tilde{\theta}_t$ , and is characterized by the indifference of the marginal borrower, given explicitly as follows:

$$\mathbb{E}[v_b(\chi(1 + \psi_{t,t+1}) + y - r_{b,t})] = \mathbb{E}[\mathcal{V}_b^{DLP}(b_t) \mid \tilde{\theta}_t] \quad (3.13)$$

---

<sup>4</sup>We define  $\mathcal{C}$  explicitly in Appendix A; Lemma EC.6 establishes the closure property.

<sup>5</sup>Default in our pseudonymous DLP entails forfeiting posted collateral, whereas default at the outside venue exposes the borrower's broader wealth to legal claims. Such recourse and the costs of bankruptcy strongly discourage strategic default at the outside venue. To ease exposition, we abstract from the details of that decision and capture the resulting incentive to repay by assuming certain repayment.

Given that  $v_b$  is strictly increasing, the left side of Equation (3.13) is strictly decreasing in  $r_{b,t}$ , so the equation admits at most one solution. We establish existence subsequently as part of the equilibrium characterization in Section 4. Then, since all borrowers  $(i, t)$  with outside borrowing rate  $r_{b,(i,t)} \geq r_{b,t}$  borrow from the DeFi lending platform, the period  $t$  borrower demand for the DLP,  $\mathcal{D}_t$ , is given explicitly as follows:

$$\mathcal{D}_t = \mu_t \cdot (1 - F_b(r_{b,t}(\tilde{\theta}_t))) \quad (3.14)$$

### 3.2 Decentralized Lending Protocol (DLP)

A DLP is defined by a single exogenous borrower interest rate function which we denote by  $\rho : [0, 1] \mapsto \mathbb{R}_+$ . Following practice, we assume that  $\rho$  is continuous and strictly increasing. Additionally, to rule out a trivial equilibrium where no borrowing ever occurs, we impose that the lowest possible borrower interest rate,  $\rho(0)$ , is low enough to ensure that some borrower would be willing to borrow at that rate. Moreover, to rule out the possibility that borrowing demand exceeds available supply of lending funds, we also impose that the highest borrowing rate,  $\rho(1)$ , is sufficiently high that no borrower would borrow at that level.<sup>6</sup> Of note,  $\rho(0) = 0$  is maintained in practice where the borrowing rate at zero utilization is typically 0% annualized while the borrowing rate at full utilization generally exceeds 50% annualized. For exposition, we hereafter refer to the set of interest rate functions that we study by  $\mathcal{IR}$ .

As noted,  $\rho$  is a borrower interest rate function. More explicitly, the DLP mechanically sets the borrower's interest rate in period  $t$ ,  $b_t$ , as follows:

$$b_t = \rho(U_t) \quad (3.15)$$

where  $U_t$  is the utilization rate in period  $t$ , given as follows:

$$U_t = \frac{\text{Total Borrowing}}{\text{Total Lending}} = \frac{\mathcal{D}_t}{\mathcal{S}_t} \quad (3.16)$$

In turn, the DLP sets the lender rate by passing realized borrower payments on borrowed capital directly to lenders while also returning unborrowed capital. With collateral, realized borrower payments depend on whether borrowers repay: each borrower repays her loan if and only if the collateral value at maturity covers her repayment obligation (i.e.,  $\chi(1 + \psi_{t,t+1}) \geq 1 + b_t$ ); otherwise she defaults and the DLP seizes her collateral. For exposition, we have assumed that the initial collateral asset value and its return are common across borrowers,

---

<sup>6</sup>Explicitly, the first condition is  $\rho(0) = 0$  and the second condition is  $\rho(1) > \chi(1 + \bar{\psi}) - 1$ .

implying that either all period  $t$  DLP borrowers repay or all default. Thus, the DLP's gross receipt per unit borrowed,  $\Phi(U_t, \psi_{t,t+1})$ , is given as follows:

$$\Phi(U, \psi) := \min\{1 + \rho(U), \chi(1 + \psi)\} \quad (3.17)$$

As noted, the lender's interest rate,  $l_t$ , is the interest rate that distributes to lenders the realized borrower payments together with their idle capital:

$$\underbrace{\mathcal{S}_t \cdot (1 + l_t)}_{\text{Payments To Lenders}} = \underbrace{\mathcal{D}_t \cdot \Phi(U_t, \psi_{t,t+1})}_{\text{Payments From Borrowers}} + \underbrace{(\mathcal{S}_t - \mathcal{D}_t)}_{\text{Idle Capital}} \quad (3.18)$$

Equations (3.16) and (3.18) then imply that the lender rate is a mechanical function of the utilization rate and the collateral return as follows:

$$l_t = l(U_t, \psi_{t,t+1}), \quad l(U, \psi) := U \cdot (\Phi(U, \psi) - 1) \quad (3.19)$$

### 3.3 Equilibrium

Given an interest rate function,  $\rho \in \mathcal{IR}$ , a DeFi lending equilibrium is a tuple of state-contingent functions: borrower and lender cut-off rates,  $r_{b,t}^* = r_b^*(\tilde{\theta}_t)$  and  $r_{l,t}^* = r_l^*(\tilde{\theta}_t)$ , a utilization rate,  $U_t^* = U^*(\tilde{\theta}_t, \varepsilon_t^+)$ , a borrower interest rate,  $b_t^* = b^*(\tilde{\theta}_t, \varepsilon_t^+)$ , and a lender interest rate,  $l_t^* = l^*(\tilde{\theta}_t, \varepsilon_t^+, \psi_{t,t+1})$ , such that, for all realizations of the state: (i) the borrower cut-off rate satisfies the marginal borrower's indifference condition, Equation (3.13); (ii) the lender cut-off rate satisfies the marginal lender's indifference condition, Equation (3.8); (iii) the utilization rate clears the market, Equations (3.9), (3.14) and (3.16); and (iv) the DLP sets interest rates mechanically, Equations (3.15) and (3.19). Of note, solving for such an equilibrium reduces to solving for the stochastic process of utilization rates. This is because, as per Equations (3.15) and (3.19), equilibrium interest rates are uniquely determined by the utilization rate and the exogenous collateral return. Moreover, as per Equations (3.8) and (3.13), cut-off interest rates are determined by the distribution of lender and borrower interest rates and by the exogenous state variable,  $\tilde{\theta}_t$ , where the former is determined by the utilization rates and the collateral return.

To determine equilibrium utilization rates, we begin with Equation (3.16) and apply Equations (3.9) and (3.14), yielding:

$$U_t = \theta_t \cdot \frac{1 - F_b(r_{b,t}(\tilde{\theta}_t))}{F_l(r_{l,t}(\tilde{\theta}_t))} \quad (3.20)$$

Subsequently, in Section 4, we establish existence and uniqueness of equilibrium by reducing the equilibrium conditions to a one-dimensional fixed point problem.

## 4 Existence and Uniqueness of a DeFi Lending Equilibrium

As established in Section 3.3, solving for a DeFi lending equilibrium reduces to determining the utilization process. To that end, for any equilibrium, we define  $\Gamma^*$  as the equilibrium ratio of the borrowing and lending participation shares, given explicitly as follows:

$$\Gamma^*(\tilde{\theta}) := \frac{1 - F_b(r_b^*(\tilde{\theta}))}{F_l(r_l^*(\tilde{\theta}))} \quad (4.1)$$

so that, given  $\theta_t = \tilde{\theta}_t \cdot \varepsilon_t^+$ , Equation (3.20) implies that the equilibrium utilization rate is given as follows:

$$U_t^* = \tilde{\theta}_t \cdot \varepsilon_t^+ \cdot \Gamma^*(\tilde{\theta}_t) \quad (4.2)$$

In turn, characterizing equilibrium amounts to characterizing the function  $\Gamma^*$ . For that purpose, it is useful to define the threshold value of  $\Gamma$  at which utilization would reach unity in the highest demand state, given explicitly as follows:

$$\bar{\Gamma}_{\tilde{\theta}} := \frac{1}{\tilde{\theta} \cdot \varepsilon_{\max}^+} \quad (4.3)$$

We now parameterize the objects that determine equilibrium by the pair  $(\tilde{\theta}, \Gamma)$ , treating  $\Gamma$  as a candidate value for  $\Gamma^*(\tilde{\theta})$ . First, given that Equation (3.19) sets the lender rate as a function of the utilization rate and the collateral return, the parameterized lender rate is given as follows:

$$l(\tilde{\theta}, \Gamma, \varepsilon^+, \psi) := l(\tilde{\theta} \varepsilon^+ \Gamma, \psi) \quad (4.4)$$

Second, the two sides of the borrower indifference condition, Equation (3.13), become the borrower's outside option value,  $L$ , and her value of borrowing via the DLP,  $R$ , given explicitly as follows:

$$L(r) := \mathbb{E}_{\psi} [v_b(\chi(1 + \psi) + y - r)] \quad (4.5)$$

$$R(\tilde{\theta}, \Gamma) := \mathbb{E}_{\psi, \varepsilon^+} [\max\{v_b(\chi(1 + \psi) + y - \rho(\tilde{\theta} \varepsilon^+ \Gamma)), v_b(1 + y)\}] \quad (4.6)$$

Finally, the parameterized cut-off rates solve the indifference conditions, Equations (3.13) and (3.8), at  $(\tilde{\theta}, \Gamma)$ , given explicitly as follows:

$$r_b(\tilde{\theta}, \Gamma) := L^{-1}(R(\tilde{\theta}, \Gamma)) \quad (4.7)$$

$$r_l(\tilde{\theta}, \Gamma) := v_l^{-1}(\mathbb{E}_{\varepsilon^+, \psi}[v_l(1 + l(\tilde{\theta}, \Gamma, \varepsilon^+, \psi))]) - 1 \quad (4.8)$$

where  $L$  is continuous and strictly decreasing so that  $L^{-1}$  is well defined on the relevant range, as we establish in the online appendix (Lemma EC.2).

Substituting Equations (4.7) and (4.8) into Equation (4.1), the equilibrium ratio  $\Gamma^*(\tilde{\theta})$  must solve the following one-dimensional fixed point problem:

$$\Gamma^*(\tilde{\theta}) = \frac{1 - F_b(r_b(\tilde{\theta}, \Gamma^*(\tilde{\theta})))}{F_l(r_l(\tilde{\theta}, \Gamma^*(\tilde{\theta})))} \quad (4.9)$$

Define the normalized excess of utilized lending supply over borrowing demand,  $\Delta_{\tilde{\theta}}$ , as follows:

$$\Delta_{\tilde{\theta}}(\Gamma) := \Gamma \cdot F_l(r_l(\tilde{\theta}, \Gamma)) - (1 - F_b(r_b(\tilde{\theta}, \Gamma))) \quad (4.10)$$

The fixed point problem, Equation (4.9), amounts to finding a root of  $\Delta_{\tilde{\theta}}$  in  $(0, \bar{\Gamma}_{\tilde{\theta}})$ . In turn, any such root generates the full equilibrium, given explicitly as follows:

$$r_b^* = r_b(\tilde{\theta}, \Gamma^*), \quad r_l^* = r_l(\tilde{\theta}, \Gamma^*), \quad U^* = \tilde{\theta} \varepsilon^+ \Gamma^*, \quad b^* = \rho(U^*), \quad l^* = l(U^*, \psi) \quad (4.11)$$

Our first result establishes the existence and uniqueness of a stationary DeFi lending equilibrium.

**Proposition 4.1.** Existence and Uniqueness of Equilibrium

For any  $\rho \in \mathcal{IR}$  there always exists a DeFi lending equilibrium, and it is unique. Moreover, every equilibrium satisfies  $U_t^* < 1$  almost surely.

**Utilization Rates**

The equilibrium utilization rate is given as follows:

$$U_t^* = \tilde{\theta}_t \cdot \varepsilon_t^+ \cdot \Gamma^*(\tilde{\theta}_t) \quad (4.12)$$

where, for each  $\tilde{\theta}_t$ ,  $\Gamma^*(\tilde{\theta}_t)$  is the unique root of  $\Delta_{\tilde{\theta}_t}$  (Equation (4.10)) and lies in  $(0, \bar{\Gamma}_{\tilde{\theta}_t})$ .

**DLP Interest Rates**

The equilibrium DLP interest rates are given as follows:

$$b_t^* = \rho(U_t^*), \quad l_t^* = l(U_t^*, \psi_{t,t+1}) \quad (4.13)$$

### Cut-off Interest Rates

The equilibrium cut-off rates are given as follows:

$$r_{b,t}^* = r_b(\tilde{\theta}_t, \Gamma^*(\tilde{\theta}_t)) \quad (4.14)$$

$$r_{l,t}^* = r_l(\tilde{\theta}_t, \Gamma^*(\tilde{\theta}_t)) \quad (4.15)$$

To demonstrate some of the features of the DeFi lending equilibrium, we first note that while users are informed about the state of the credit market at the beginning of period  $t$  (i.e.,  $\tilde{\theta}_t$ ), they are uninformed about the eventual state of the credit market that is realized at the end of the period after borrowing and lending decisions are made (i.e.,  $\theta_t = \varepsilon_t^+ \cdot \tilde{\theta}_t$ ). Further, as demonstrated by Proposition 4.1, it is the state at the end of the period,  $\theta_t$ , that determines the interest rate that borrowers pay,  $b_t^*$ , while the interest rate that lenders receive,  $l_t^*$ , depends additionally on the realized collateral return,  $\psi_{t,t+1}$ . Thus, users make their borrowing and lending decisions under uncertainty, understanding that the intermediate shock  $\varepsilon_t^+$  will affect the state of the credit market,  $\theta_t$ , and thereby also affect the realized utilization rates and realized interest rates. More formally, Equations (4.12) and (4.13) demonstrate that the utilization rate  $U_t^*$ , the borrowing rate  $b_t^*$ , and the lending rate  $l_t^*$  all depend upon credit market tightness,  $\theta_t$ . However, since users cannot perfectly anticipate credit market tightness, they make their decisions to maximize expected utility given the uncertainty about borrowing and lending rates. In turn, Equations (4.14) and (4.15) demonstrate that the equilibrium cut-off rates depend upon the best information available at the beginning of period  $t$ , namely  $\tilde{\theta}_t$  rather than  $\theta_t$ , which is revealed only after decisions are made.

To understand why existence and uniqueness of equilibrium holds generically, it is useful to specialize to the case that the intermediate shock  $\varepsilon_t^+$  is deterministic (i.e., there exists  $\hat{\varepsilon} \in \mathbb{R}_+$  such that  $\pi_+$  places unit mass at  $\hat{\varepsilon}$ ). In this case, borrowers and lenders perfectly anticipate the utilization rate,  $U_t$ , and their decisions can therefore be written directly as a function of the perfectly anticipated utilization rate (i.e.,  $r_{b,t}$  and  $r_{l,t}$  are functions of  $U_t$ ). Then, since borrower and lender decisions are functions of the utilization rate, lending supply and borrowing demand are also therefore each a function of the utilization rate (i.e.,  $\mathcal{D}_t$  and  $\mathcal{S}_t$  are functions of  $U_t$ ). Importantly, because borrower interest rates and lender interest rates both mechanically increase in the utilization rate (see Equations (3.15) and (3.19)), borrowing demand necessarily decreases in the utilization rate (i.e.,  $\mathcal{D}_t$  decreases in  $U_t$ ) whereas lending supply necessarily increases in the utilization rate (i.e.,  $\mathcal{S}_t$  increases in  $U_t$ ). Further, recall that solving for a DeFi lending equilibrium reduces to determining the DLP utilization

rate. As a consequence, the DeFi lending equilibrium reduces to finding a utilization rate  $U_t$ , a lending supply  $\mathcal{S}_t$  and a borrowing demand  $\mathcal{D}_t$  which are self-consistent. More concretely, market clearing requires that the utilized lending supply implied by the utilization rate,  $U_t \cdot \mathcal{S}_t$ , equate with the borrowing demand,  $\mathcal{D}_t$ . Crucially, as discussed, lending supply increases in  $U_t$  so that the utilized lending supply also increases in  $U_t$ , whereas borrowing demand decreases in  $U_t$ . Therefore, the excess of utilized lending supply beyond borrowing demand is strictly increasing in the utilization rate, and there is consequently a unique utilization rate at which it vanishes, which is then necessarily the unique equilibrium utilization rate  $U_t^*$ . This argument is formalized by the excess supply function  $\Delta_{\tilde{\theta}_t}$  in Equation (4.10): up to a positive scaling,  $\Delta_{\tilde{\theta}_t}(\Gamma)$  measures the excess of utilized lending supply over borrowing demand along candidate utilization  $U_t = \tilde{\theta}_t \varepsilon_t^+ \Gamma$ , and the unique equilibrium of Proposition 4.1 obtains precisely from its unique root.

Intuitively, the utilization rate plays a similar role for the DLP interest rate protocol as the market-clearing rate plays in a competitive equilibrium. In particular, in both settings, borrower demand slopes downward whereas lending supply slopes upward in the endogenous quantity that determines equilibrium. In the DLP case, that endogenous quantity is the utilization rate whereas, in the competitive equilibrium setting, that endogenous quantity is the market-clearing interest rate. Notably, Proposition 4.1 generalizes the intuition stated above to apply also when the borrowers and lenders do not know the end-of-period utilization rate directly and instead act based on the anticipated distribution of end-of-period utilization rates under the condition that this distribution must be consistent with their lending and borrowing actions in equilibrium. In that generalized context, Proposition 4.1 establishes a unique stationary equilibrium.

The key implication of our existence and uniqueness result is that DLP lending and borrowing activity fluctuates naturally with market conditions. In turn, while adverse market conditions might lead to reduced economic activity, DLP economic activity would nonetheless revert with improvements in market conditions. Practically speaking, our results help explain how DLPs have survived despite significant fluctuations in credit market conditions.

## 5 DLP Inefficiencies

Having established uniqueness of equilibrium, we can now unambiguously examine the efficiency properties of a DLP. To understand the subsequent analysis, it is first important to understand that there are two sources of uncertainty about the state of the credit market within each period:  $\varepsilon_t^-$  and  $\varepsilon_t^+$ . The first source of uncertainty,  $\varepsilon_t^-$ , is known to users when they choose their borrowing and lending venues, whereas the second source of uncertainty,

$\varepsilon_t^+$ , is not known to users at that time. In contrast, neither source of uncertainty is explicitly incorporated into the DLP interest rate setting procedure (i.e., Equations (3.15)–(3.19)) due to a technical constraint known as the oracle problem (see John et al. 2023). Notably, one might expect that both sources of DLP uncertainty contribute to the inefficiency of interest rates set by the DeFi lending platform. However, we will demonstrate that only the latter source of uncertainty precludes the DeFi lending platform from setting efficient interest rates in each period. In particular, if there is no user uncertainty about the state of the credit market (e.g., if  $\varepsilon_t^+$  were deterministic) then the fluctuations in the state of the credit market — observable to the users but not the DLP — do not affect the DLP’s ability to set market-clearing interest rates when the DLP uses a properly designed interest rate function. In contrast, if borrowers and lenders face uncertainty about the state of the credit market, then the DLP sets inefficient interest rates, no matter the interest rate function of the DLP. Thus, the key inefficiency of any lending platform that utilizes this form of interest rate setting mechanism arises from user uncertainty about the state of the credit market and not from the inherent inability of the DLP to condition on recent information (i.e., the oracle problem).

To provide some intuition for why  $\varepsilon_t^+$  (but not  $\varepsilon_t^-$ ) drives inefficiency, it is important to recognize that the utilization rate,  $U_t^*$ , is endogenous and that the DLP’s interest rate setting procedure depends on this endogenous utilization rate. In turn, when there is no user uncertainty regarding market tightness (i.e.,  $\varepsilon_t^+$  is deterministic), then the utilization rate  $U_t^*$  is determined entirely by the start of period  $t$  credit market condition,  $\tilde{\theta}_t$  (see Equation (4.12)). Crucially, in that case, a well-specified interest rate function can effectively ensure that equilibrium utilization rates are as close to unity (i.e., market clearing) as desired by harnessing the users’ knowledge of the state of the credit market. Intuitively, information known to users (i.e.,  $\tilde{\theta}_t$ ) can be approximately inferred by the DLP through observing economic outcomes (i.e.,  $U_t^*$ ) whenever those economic outcomes are fully determined by user actions. In contrast, when the observed economic outcomes (i.e.,  $U_t^*$ ) depend also on exogenous shocks not perfectly known to users, then inferring credit market conditions from observed economic outcomes is no longer feasible.

## 5.1 Benchmark: No User Uncertainty About the State of the Credit Market

In order to proceed, we will first study the benchmark case whereby the intermediate shock  $\varepsilon_t^+$  is deterministic (i.e., there exists  $\hat{\varepsilon} \in \mathbb{R}_+$  such that  $\pi_+$  places unit mass at  $\hat{\varepsilon}$ ), implying that  $\theta_t$  is known at the start of period  $t$  before borrowing and lending decisions are made.

In this case, all borrowing and lending decisions can be written as a function of  $\theta_t$  directly by using  $\theta_t = \tilde{\theta}_t \cdot \hat{\varepsilon}$ . This benchmark case allows us to study the optimal design of the DLP interest rate function when users (but not the DLP) are perfectly informed about the state of the credit market at the time when their borrowing and lending decisions are made. Note, however, that even in this benchmark users are not free of all uncertainty: the collateral return,  $\psi_{t,t+1}$ , remains random, so that while borrowers can perfectly anticipate the DLP borrowing rate that they will pay, the realized DLP lending rate remains risky.

To proceed, note that when  $\varepsilon_t^+$  is deterministic then Borrower  $(i, t)$  can perfectly anticipate the borrowing rate,  $b_t^* = \rho(U_t^*)$ , that she will pay at the DLP. Unlike in a setting without collateral, however, a common rate does not make the two borrowing venues payoff-equivalent, because a DLP loan carries the option to default. The marginal borrower is therefore characterized by indifference between the two venues: Borrower  $(i, t)$  will optimally borrow from the DLP if and only if  $r_{b,(i,t)} \geq \hat{r}_b(U_t^*)$ , where the borrower cut-off function obtains from Equation (4.7) as follows:

$$\hat{r}_b(U) := L^{-1}(\mathbb{E}_\psi[\max\{v_b(\chi(1 + \psi) + y - \rho(U)), v_b(1 + y)\}]) \quad (5.1)$$

That is,  $\hat{r}_b(U_t^*)$  is the outside borrowing rate at which a borrower is exactly indifferent between her outside option and borrowing from the DLP at the anticipated rate  $\rho(U_t^*)$ . Of note, the option to default implies  $\hat{r}_b(U) \leq \rho(U)$ , so that borrowers whose outside rates lie below the DLP borrowing rate may nonetheless optimally borrow from the DLP. Similarly, while Lender  $(j, t)$  faces no uncertainty about the state of the credit market, the realized DLP lending rate,  $l_t^* = l(U_t^*, \psi_{t,t+1})$ , remains risky through the collateral return. Therefore, Lender  $(j, t)$  will optimally lend to the DLP if and only if  $r_{l,(j,t)} \leq \hat{r}_l(U_t^*)$ , where the lender cut-off function obtains from Equation (4.8) as the certain lending rate that delivers the same utility as the risky DLP lending rate:

$$\hat{r}_l(U) := v_l^{-1}(\mathbb{E}_\psi[v_l(1 + l(U, \psi))]) - 1 \quad (5.2)$$

Then, applying the cut-offs  $r_{b,t}^* = \hat{r}_b(U_t^*)$  and  $r_{l,t}^* = \hat{r}_l(U_t^*)$  to Equation (3.20),  $\varepsilon_t^+$  being deterministic necessarily implies that the equilibrium utilization rate in period  $t$  is given by the unique solution,  $U_t^*$ , to the following equation:

$$U_t^* = \theta_t \cdot \frac{1 - F_b(\hat{r}_b(U_t^*))}{F_l(\hat{r}_l(U_t^*))} \quad (5.3)$$

We will now proceed to our next result which states that, absent user uncertainty about the state of the credit market (i.e., when  $\varepsilon_t^+$  is deterministic), the DLP can generate efficient

interest rates by ensuring that the equilibrium utilization rate is arbitrarily close to unity in each period  $t$ .

**Proposition 5.1.** *Suppose that  $\varepsilon_t^+$  is deterministic (i.e., there exists  $\hat{\varepsilon} \in \mathbb{R}_+$  such that  $\pi_+$  places unit mass at  $\hat{\varepsilon}$ ) and denote by  $U^*(\theta_t, \rho)$  the equilibrium utilization rate when the known state of the credit market is  $\theta_t$  and the DLP utilizes the interest rate function  $\rho$ . Then, for any  $\delta > 0$ , there exists  $\rho \in \mathcal{IR}$  such that  $U^*(\theta_t, \rho) \in [1 - \delta, 1)$  for all  $\theta_t \in [\theta_{\min}, \theta_{\max}]$ , where  $\theta_{\min}$  and  $\theta_{\max}$  are defined in Equation (A.1) of Appendix A.*

Proposition 5.1 states that by using an appropriate interest rate function, the DLP can guarantee that equilibrium utilization is arbitrarily close to unity in each period. The key implication of this result is that any inefficiencies caused by the DLP interest rate setting procedure can be eliminated through the appropriate choice of the interest rate function. More explicitly, whenever  $U_t^* < 1$  then the DLP features an excess supply of capital whereby a fraction  $1 - U_t^* > 0$  of capital that is lent to the platform is not borrowed in period  $t$ . Thus,  $U_t^* < 1$  represents a key inefficiency as any idle capital could be redeployed to more productive use. The key to eliminating this waste is therefore to ensure that equilibrium utilization is as close to unity as possible in each period  $t$  which Proposition 5.1 states is possible through the appropriate choice of the interest rate function.

Proposition 5.1 is achieved by specifying a DLP borrower interest rate function that sets interest rates that are low when utilization is sufficiently below unity and prohibitively high when utilization is close to unity. Such interest rate functions avoid a low utilization equilibrium because the implied borrower interest rates under low utilization are sufficiently low that many borrowers will have a profitable deviation to increase borrowing. Further, the low borrower rates under low utilization will lead to low lender rates so that many lenders will have a profitable deviation to decrease lending with the borrower and lender deviations both leading to higher utilization. Similarly, such interest rate functions also ensure that equilibrium utilization is bounded away from unity because borrowing rates become prohibitive as utilization approaches unity. Thus, if borrowers anticipate a sufficiently high utilization rate, then many of them will have a profitable deviation to decrease borrowing. Further, if the borrower rate is large when utilization is close to unity then so will be the lender rate. In this case, high anticipated utilization will lead many lenders to have a profitable deviation to lend more to the platform with both borrower and lender deviations leading to lower utilization. In summary, any continuous interest rate function that sets sufficiently low interest rates when utilization is far from unity (e.g., below  $1 - \delta$ ) and sets sufficiently high interest rates when utilization is equal to unity will ensure that equilibrium utilization is close but never equal to unity in each period (i.e.,  $U_t^* \in [1 - \delta, 1)$ ), no matter the realized state of the credit market in each period.

We next turn to the welfare implications of Proposition 5.1. In particular, we define welfare as the integral over all agents' utilities and compare the welfare generated by the DeFi equilibrium to that of a competitive equilibrium whereby markets clear in each period (see Appendix B for precise welfare definitions). Then, a consequence of Proposition 5.1 is that the welfare generated by the DeFi equilibrium can be made arbitrarily close to the welfare of a perfectly competitive, market-clearing, equilibrium through the appropriate choice of the interest rate function as demonstrated by the following result:

**Proposition 5.2.** *Suppose that  $\varepsilon_t^+$  is deterministic (i.e., there exists  $\hat{\varepsilon} \in \mathbb{R}_+$  such that  $\pi_+$  places unit mass at  $\hat{\varepsilon}$ ). Let  $\mathcal{W}_t^{DeFi}(\rho)$  denote the DeFi equilibrium welfare when the DLP interest rate function is  $\rho$ , and let  $\mathcal{W}_t^{CE}$  denote the competitive equilibrium welfare. Then, for any  $\varepsilon_w > 0$  there exists  $\rho \in \mathcal{IR}$  such that:*

$$|\mathbb{E}[\mathcal{W}_t^{DeFi}(\rho)] - \mathbb{E}[\mathcal{W}_t^{CE}]| < \varepsilon_w \quad (5.4)$$

for all  $t \in \mathbb{N}$ , where the expectations are with respect to the stationary distributions over borrower and lender measures and the collateral return.

Proposition 5.2 states that expected DeFi welfare can be made arbitrarily close to the expected competitive equilibrium welfare through the appropriate choice of interest rate function with the expectation taken with respect to the stationary distributions over borrower and lender measures and the collateral return. Thus, Proposition 5.2 states that although the interest rate function must be set in advance at time  $t = 0$  and cannot condition on any state of the credit market (i.e., neither  $\tilde{\theta}_t$  nor  $\theta_t$ ), the DLP can still generate outcomes that are arbitrarily close to the competitive equilibrium outcomes by choosing the appropriate interest rate function when there is no user uncertainty about the state of the credit market (i.e.,  $\varepsilon_t^+$  is deterministic). The appropriate interest rate function is any function that ensures that equilibrium utilization is close but never equal to unity and the existence of such a function is guaranteed by Proposition 5.1. To see why Proposition 5.2 holds, it is important to recognize that, when equilibrium utilization is close to unity, then it must be the case that the DLP borrowing rate is close to the market-clearing interest rate and the DLP lender return is close to the collateralized claim earned in the competitive equilibrium. Moreover, when utilization rates are close to unity and the rates and returns faced by borrowers and lenders are close to their competitive equilibrium counterparts, then DeFi welfare must necessarily be close to the competitive equilibrium welfare in each period. Importantly, the DLP borrower interest rate function is set as a static function, once at inception (i.e., at  $t = 0$ ) and then not modified thereafter. Nonetheless, Proposition 5.2 arises because the DLP borrower interest rate function can be selected in such a way that the equilibrium

interest rates will be arbitrarily close to the market-clearing rates in each subsequent period  $t > 0$  irrespective of the realized state of the credit market.

To provide more detail, note that the DLP's mechanical interest rate setting rules (i.e., Equations (3.15) and (3.19)) imply that, as utilization converges to unity, the lender's return converges to the full collateralized claim on the borrower's payment (i.e.,  $1 + l_t \rightarrow \min\{1 + b_t, \chi(1 + \psi_{t,t+1})\}$  as per Equation (3.17)). To characterize the limit, we introduce, for any quoted rate  $r \geq 0$ , the full-utilization counterparts of the anticipated cut-off functions:

$$\tilde{r}_b(r) := L^{-1}(\mathbb{E}_\psi[\max\{v_b(\chi(1 + \psi) + y - r), v_b(1 + y)\}]) \quad (5.5)$$

$$\tilde{r}_l(r) := v_l^{-1}(\mathbb{E}_\psi[v_l(\min\{1 + r, \chi(1 + \psi)\}]) - 1 \quad (5.6)$$

That is,  $\tilde{r}_b(r)$  is the outside rate at which a borrower is indifferent to borrowing from the DLP at a quoted rate  $r$  (so that  $\hat{r}_b(U) = \tilde{r}_b(\rho(U))$ ), and  $\tilde{r}_l(r)$  is the certain rate that delivers the same utility as a fully utilized claim paying  $\min\{1 + r, \chi(1 + \psi)\}$ . Then, via Proposition 5.1, we construct a sequence of DLP borrower interest rate functions,  $\{\rho_n\}_{n=1}^\infty$ , such that for any state  $\theta_t$  the associated equilibrium utilization rates generated when using each function in the sequence converge to unity (i.e.,  $\lim_{n \rightarrow \infty} U_n^* = 1$ ). In turn, letting  $r^* := \lim_{n \rightarrow \infty} \rho_n(U_n^*)$  denote the limiting DLP borrowing rate, the associated equilibrium cut-offs converge to their full-utilization counterparts (i.e.,  $r_{b,n}^* \rightarrow \tilde{r}_b(r^*)$  and  $r_{l,n}^* \rightarrow \tilde{r}_l(r^*)$ ), so that taking limits on each side of Equation (5.3) yields the following:

$$1 = \theta_t \cdot \frac{1 - F_b(\tilde{r}_b(r^*))}{F_l(\tilde{r}_l(r^*))} \quad (5.7)$$

Crucially, Equation (5.7) is equivalent to requiring that the quoted rate  $r^*$  clears the credit market. More formally, by direct verification, we have the following equivalence:

$$\underbrace{1 = \theta_t \cdot \frac{1 - F_b(\tilde{r}_b(r^*))}{F_l(\tilde{r}_l(r^*))}}_{\text{DLP IR, } r^*, \text{ if Utilization} = 1} \Leftrightarrow \underbrace{\lambda_t \cdot F_l(\tilde{r}_l(r^*)) = \mu_t \cdot (1 - F_b(\tilde{r}_b(r^*)))}_{\text{Lending Supply} = \text{Borrowing Demand at } r^*} \quad (5.8)$$

Notably, the right-hand side of Equation (5.8) imposes market clearing at the quoted rate  $r^*$ , which is the definition of a competitive equilibrium interest rate in our setting (see Appendix B). Thus,  $r^*$  in Equation (5.8) is the competitive equilibrium interest rate: the DLP borrowing rate converges to  $r^*$ , and the borrower and lender cut-offs converge to their competitive equilibrium counterparts,  $\tilde{r}_b(r^*)$  and  $\tilde{r}_l(r^*)$ . Crucially, social welfare is a continuous function of the interest rates and cut-offs faced by borrowers and lenders. Consequently, the DLP welfare necessarily converges to the competitive equilibrium welfare as  $n \rightarrow \infty$ .

We have just demonstrated that the DLP can overcome its own uncertainty about the

state of the credit market by harnessing the platform users' knowledge of that state and their incentives to borrow and lend. In doing so, the DLP is able to produce utilization rates that are arbitrarily close to unity and, therefore, arbitrarily close to market clearing. Thus, when there is no uncertainty faced by DLP users about the state of the credit market then the platform can generate welfare that is arbitrarily close to the competitive equilibrium welfare through the appropriate choice of the interest rate function. In such a context, the utilization rate (in tandem with the interest rate function) effectively reveals the state of the credit market to the DLP through the equilibrium cut-offs that are optimally chosen by borrowers and lenders. In contrast, as we will demonstrate in the next section, once users face uncertainty about the state of the credit market, the utilization rate no longer reveals the state of the credit market. In that case, even if the DLP could learn the information of its users about the state of the credit market at the start of period  $t$ , the uncertainty of borrowing and lending interest rates will prevent the DLP from setting market-clearing interest rates, no matter the choice of the interest rate function.

## 5.2 User Uncertainty About the State of the Credit Market

In this section we will study the case whereby  $\varepsilon_t^+$ , the shock that occurs to the state of the credit market after borrowing and lending decisions are made, is no longer deterministic and therefore users face uncertainty about the state of the credit market when they make their borrowing and lending decisions. Recall that the key driver of the results of Proposition 5.1, which states that the inefficiencies of excess DLP supply can be eliminated, is the fact that borrowers and lenders know the relevant state of the credit market,  $\theta_t$ , when making their decisions. This feature, imposed when assuming that  $\varepsilon_t^+$  is deterministic, ensures that borrowers and lenders can perfectly anticipate the equilibrium utilization rate and hence the DLP borrowing rate, with the realized DLP lending rate varying only with the exogenous collateral return. In contrast, when  $\varepsilon_t^+$  is non-deterministic, borrowers and lenders know the start of period  $t$  state of the credit market,  $\tilde{\theta}_t$ , but they face uncertainty about the end of period  $t$  state of the credit market,  $\theta_t$ , that determines the realized borrowing rate and, together with the realized collateral return, the realized lending rate. We will show that this uncertainty is sufficient to guarantee that realized equilibrium DeFi utilization is uniformly bounded away from unity with positive probability, no matter the interest rate function employed by the DLP. Thus, the DLP will always feature excess supply with positive probability no matter the interest rate function, leading to capital that is lent to the platform but not borrowed. Our next result formalizes this point.

**Proposition 5.3.** *For any  $\varepsilon^+ \in S_+$ , denote by  $U^*(\tilde{\theta}_t, \varepsilon^+, \rho)$  the realized equilibrium uti-*

lization rate when the realized state of the credit market is  $\theta_t = \tilde{\theta}_t \cdot \varepsilon^+$  and the DLP utilizes the interest rate function  $\rho$ . If  $\varepsilon_t^+$  is not deterministic (i.e.,  $|S_+| > 1$ ), then for all  $\varepsilon^+ \neq \varepsilon_{\max}^+ := \max S_+$  there exists  $\delta > 0$  such that  $U^*(\tilde{\theta}_t, \varepsilon^+, \rho) < 1 - \delta$  uniformly for all  $\rho \in \mathcal{IR}$  and all  $\tilde{\theta}_t$ .

Proposition 5.3 states that whenever investors face uncertainty about the state of the credit market then the equilibrium utilization rate at the DLP must be uniformly lower than unity for all but the largest realization of the period  $t$  shock  $\varepsilon_t^+$ , no matter the DLP borrower interest rate function. In turn, the DeFi lending platform will feature excess supply with positive probability no matter the interest rate function, representing a key inefficiency of the DeFi lending platform. This result holds because the dispersion in the ex-post shock,  $\varepsilon_t^+$ , necessarily translates to a dispersion in the realized utilization rate. Notably, even if utilization is arbitrarily close to unity for the largest realization of the credit market shock  $\varepsilon_t^+$ , utilization must be strictly lower than unity for all other smaller realizations of the shock  $\varepsilon_t^+$ . Therefore, there must exist a positive probability that utilization rates are bounded away from unity and we show that this bound is uniform across all DLP borrower interest rate functions.

To provide more detail, let  $U_{t,h}^*$  denote the realized equilibrium utilization rate generated when the shock  $\varepsilon_t^+$  is equal to the highest value in its support  $S_+$ , which we denote by  $\varepsilon_h^+ := \max S_+$ . Similarly denote by  $U_{t,l}^*$  the realized equilibrium utilization rate generated when the shock  $\varepsilon_t^+$  is equal to any value  $\varepsilon_l^+ \in S_+$  such that  $\varepsilon_l^+ < \varepsilon_h^+$ . Then, Equation (4.12) implies:

$$U_{t,l}^* = \frac{\varepsilon_l^+}{\varepsilon_h^+} \cdot U_{t,h}^* \quad (5.9)$$

with  $\varepsilon_h^+ > \varepsilon_l^+$  holding by definition. Crucially, Proposition 4.1 establishes that equilibrium utilization satisfies  $U_t^* < 1$  with probability one, no matter the interest rate function. The underlying economics are intuitive. Strategic default is most tempting at high utilization rates, when required repayments are large relative to the value of the posted collateral. To dissuade such default, the DLP requires the collateral to be of sufficiently high quality (formalized by the stochastic dominance condition on collateral in Section 3.1.2). High-quality collateral, however, is costly to surrender. A borrower who anticipates defaulting with high likelihood (precisely the situation at high utilization rates) therefore prefers not to borrow at all. Equilibrium utilization consequently remains below the highest feasible utilization rates. In particular, the highest realization of utilization satisfies  $U_{t,h}^* < 1$ . In turn, Equation (5.9) implies:

$$U_{t,l}^* < \frac{\varepsilon_l^+}{\varepsilon_h^+} =: 1 - \delta \quad (5.10)$$

where  $\delta$  is defined by  $\delta := 1 - \frac{\varepsilon_l^+}{\varepsilon_h^+}$  and  $\delta > 0$  necessarily follows from  $\varepsilon_h^+ > \varepsilon_l^+$ . Thus, with user uncertainty, the DLP must generate utilization rates bounded uniformly away from unity with strictly positive probability (i.e.,  $\delta > 0$  does not depend on the DLP interest rate function). In turn, realized interest rates must be uniformly bounded away from market-clearing rates no matter the interest rate function utilized.

## 6 Modern DeFi Lending Protocols

In this section we relate our results to the practice of DeFi lending. We show that the deployed interest rate mechanism of Morpho, one of the most prominent DLPs, is consistent with the guidance of our results. Specifically, Morpho targets a utilization rate and dynamically adjusts its curve in a manner similar to our guidance to achieve the target utilization. Additionally, we show that Morpho’s recent fixed-payment protocol complements rather than replaces the pooled variable-rate design that we study.

The core insight from Section 5.1 is that, when users are perfectly informed about the state of the credit market, dynamic adjustments in the DLP interest rate function can generate utilization rates arbitrarily close to a target rate. More concretely, Proposition 5.1 is established through a sequence of interest rate functions under which endogenous utilization rates become arbitrarily close to unity, the target rate at which DLP outcomes match competitive lending equilibrium outcomes. Hereafter, we clarify that this same insight is applied by the Morpho lending protocol where the interest rate function similarly adjusts to bring utilization close to a target rate. We begin by clarifying the details of the Morpho protocol, focusing on how the interest rate function adjusts. We then explain how these adjustments match our own sequence of interest rate functions from Propositions 5.1 and 5.2.

Morpho’s lending markets specify their interest rate function as follows:<sup>7</sup>

$$\rho_t^M(U) = r_t^M \cdot \begin{cases} 1 + (C - 1) \cdot \frac{U - U^M}{1 - U^M}, & U \in (U^M, 1] \\ 1 + \left(1 - \frac{1}{C}\right) \cdot \frac{U - U^M}{U^M}, & U \in [0, U^M] \end{cases} \quad (6.1)$$

where  $U^M \in (0, 1)$  is the target utilization rate,  $C > 1$  is a static quantity, and  $r_t^M$  is a quantity that updates to allow for targeting of utilization. To add context, at every instant,

---

<sup>7</sup>Morpho’s deployed mechanism, the *AdaptiveCurveIRM*, is employed by nearly all of Morpho’s markets and is documented at <https://docs.morpho.org>; the deployed contract is viewable at <https://etherscan.io/address/0x870aC11D48B15DB9a138Cf899d20F13F79Ba00BC>.

the protocol raises  $r_t^M$  when utilization runs above the target utilization and lowers  $r_t^M$  when utilization runs below it. Over time, the mechanism thus produces a sequence of interest rate functions. With the target utilization rate fixed, this sequence of functions resembles the sequence of interest rate functions from Propositions 5.1 and 5.2.

To expand further, given that our aim is to match the competitive equilibrium, which features full utilization, our target corresponds to  $U^M = 1$ . Thus, with  $U < U^M = 1$ , as holds with probability one in every equilibrium (Proposition 4.1), Morpho’s algorithm would consistently reduce  $r_t^M$ , thereby flattening the interest rate curve. This flattening is precisely the operation that our sequence performs. In particular, along the sequence of interest rate functions in Propositions 5.1 and 5.2, each successive function lowers the interest rate at every utilization level below the target while the rate at the target itself remains prohibitively high. In turn, the cheaper borrowing raises equilibrium utilization toward the target, and iterating this adjustment drives utilization arbitrarily close to unity (Proposition 5.1) while driving welfare arbitrarily close to the competitive equilibrium welfare (Proposition 5.2). Thus, the adjustment that Morpho’s algorithm prescribes whenever utilization runs below its target qualitatively matches the adjustment through which our analysis attains the competitive benchmark: flatten the interest rate curve until utilization reaches the target.

Notably, Morpho does not implement our exact algorithm, under which the curve flattens arbitrarily so that the interest rate at the target becomes arbitrarily large relative to every rate below it. This is precisely because our results in Section 5.1 hold under the assumption that users face no uncertainty about the state of the credit market, whereas users do face such uncertainty in practice. With an aggressively flattened curve, that uncertainty translates small differences between anticipated and realized utilization into large swings in realized interest rates. In Section 5.2, we formalize this limitation: when users face uncertainty, utilization remains bounded away from unity with positive probability, no matter the interest rate function employed (Proposition 5.3), so that our approach cannot fully succeed. In turn, a practical approach is to apply our insight incrementally: adjust the curve slightly across time to move utilization toward the desired level. As discussed, this is the approach employed by Morpho, whose deployed markets target  $U^M = 0.9$  rather than full utilization.

To add further context regarding onchain lending, we note that Morpho’s most recent protocol, Morpho Midnight, launched in July 2026 and adds fixed-payment lending alongside the variable-rate markets discussed above.<sup>8</sup> A Midnight loan is a zero-coupon contract: the borrower’s obligation, one unit of the loan asset per unit of debt at maturity, is fixed at origination, and the rate is implicit in the price at which lenders’ and borrowers’ offers

---

<sup>8</sup>“Now Live: Morpho Midnight & New Markets App,” Morpho, 21 July 2026, <https://morpho.org/blog/now-live-morpho-midnight>.

execute. Midnight accordingly has no utilization-based interest rate function, because such a structure would imply floating interest rates whereas Midnight specifically aims to fix the borrower obligation. Lending in Midnight nonetheless is not bilateral. Rather, it remains pooled, as in the lending protocols that we study (Saleh and Travers 2026). Morpho itself presents the two protocols as the fixed and floating sides of a single lending network. Indeed, Morpho’s co-founder and chief executive, Paul Frambot, describes Morpho’s pooled markets as “pool-based open term variable markets” and Midnight as “intent-based fixed term fixed rate markets,” adding that “the two will coexist, complementing one another to extend the capabilities of the Morpho network.”<sup>9</sup> Just as fixed-rate instruments complement rather than displace floating-rate instruments in traditional markets, Midnight complements the pooled utilization-based markets, which remain the core design for variable-rate DeFi lending and the object of our analysis.

## 7 Conclusion

In this paper, we study the interest rate setting mechanism of a Decentralized Lending Protocol (DLP). A DLP sets an interest rate function that determines the borrowing and lending rates in each subsequent period as a mechanical function of the utilization of funds (i.e., the ratio of borrowed to available funds). We demonstrate that despite the use of this mechanical procedure, DLPs ensure a unique stationary DeFi lending equilibrium under general conditions. We then turn to understanding the inefficiencies incurred by DLPs due to the use of these simple interest rate setting rules. We demonstrate that DLPs can set efficient interest rates if users are informed about the state of the credit market when making their borrowing and lending decisions. In contrast, we demonstrate that the key inefficiency produced by the DLP interest rate setting procedure arises solely when users face uncertainty about the state of the credit market. In this case, we show that the DLP will be unable to set efficient interest rates, no matter the interest rate function utilized. These results generate important implications for the design and use of DeFi lending platforms in practice.

---

<sup>9</sup>P. Frambot, 14 April 2026, <https://x.com/PaulFrambot/status/2044024001719140377>.

## References

- Aramonte S, Doerr S, Huang W, Schrimpf A, et al. (2022) DeFi lending: intermediation without information? Technical report, Bank for International Settlements.
- Biais B, Bisière C, Bouvard M, Casamatta C (2019) The blockchain folk theorem. *Review of Financial Studies* 32(5):1662–1715.
- Cao D, Falk BH, Kogan L, Tsoukalas G (2025) A structural model of automated market making, URL <http://dx.doi.org/10.2139/ssrn.4591447>, working paper.
- Capponi A, Jia R (2025) Liquidity provision on blockchain-based decentralized exchanges. *Review of Financial Studies* 38(10):3040–3085, URL <http://dx.doi.org/10.1093/rfs/hhaf046>.
- Carré S, Gabriel F (2022) Security and efficiency in DeFi lending. *Working Paper*.
- Chiu J, Ozdenoren E, Yuan K, Zhang S (2022) On the fragility of DeFi lending. *Available at SSRN 4328481*.
- Cong LW, Li Y, Wang N (2021) Tokenomics: Dynamic adoption and valuation. *Review of Financial Studies* 34(3):1105–1155.
- Cong LW, Prasad ES, Rabetti D (2026) Financial and informational integration through oracle networks. *Review of Financial Studies*, forthcoming.
- Coval J, Stafford E (2007) Asset fire sales (and purchases) in equity markets. *Journal of Financial Economics* 86(2):479–512.
- d’Avernas A, Maurin V, Vandeweyer Q (2026) Can stablecoins be stable? *Management Science*, URL <http://dx.doi.org/10.1287/mnsc.2024.06992>, Articles in Advance.
- Dong T, Laturnus V, Zhang H (2025) How do shareholder defaults influence corporate governance? *Working Paper*.
- Duffie D (2010) Presidential address: Asset price dynamics with slow-moving capital. *Journal of Finance* 65(4):1237–1267.
- Easley D, O’Hara M, Basu S (2019) From mining to markets: The evolution of Bitcoin transaction fees. *Journal of Financial Economics* 134(1):91–109.
- Geanakoplos J (2010) The leverage cycle. *NBER Macroeconomics Annual 2009*, volume 24, 1–65 (University of Chicago Press).

- Harrison JM, Kreps DM (1979) Martingales and arbitrage in multiperiod securities markets. *Journal of Economic Theory* 20(3):381–408.
- Hasbrouck J, Rivera TJ, Saleh F (2026a) An economic model of a decentralized exchange with concentrated liquidity. *Management Science* 72(5):3666–3683, URL <http://dx.doi.org/10.1287/mnsc.2024.04510>.
- Hasbrouck J, Rivera TJ, Saleh F (2026b) The need for fees at a DEX: How increases in fees can increase DEX trading volume. *Management Science*, URL <http://dx.doi.org/10.1287/mnsc.2023.00726>, Articles in Advance.
- John K, Kogan L, Saleh F (2023) Smart contracts and decentralized finance. *Annual Review of Financial Economics* 15(1):523–542, URL <http://dx.doi.org/10.1146/annurev-financial-110921-022806>.
- John K, Rivera T, Saleh F (2025) Proof-of-work versus proof-of-stake: A comparative economic analysis. *Review of Financial Studies* 38(7).
- John K, Saleh F (2025) Cryptoeconomics. *Oxford Research Encyclopedia of Economics and Finance* (Oxford University Press), URL <http://dx.doi.org/10.1093/acrefore/9780190625979.013.956>.
- Lehar A, Parlour C (2025) Decentralized exchange: The Uniswap automated market maker. *Journal of Finance* 80(1):321–374, URL <http://dx.doi.org/10.1111/jofi.13405>.
- Lehar A, Parlour CA (2023) Systemic fragility in decentralised markets. BIS Working Papers 1062, Bank for International Settlements.
- Li Y, Mayer S (2022) Money creation in decentralized finance: A dynamic model of stablecoin and crypto shadow banking. *Working Paper*.
- Mayer S (2022) Token-based platforms and speculators. *Working Paper*.
- Mueller P (2022) DeFi leveraged trading: Inequitable costs of decentralization. *Available at SSRN 4241356*.
- Park A (2023) The conceptual flaws of decentralized automated market making. *Management Science* 69(11):6731–6751, URL <http://dx.doi.org/10.1287/mnsc.2021.02802>.
- Ross SA (1978) A simple approach to the valuation of risky streams. *Journal of Business* 51(3):453–475.

- Rosu I, Saleh F (2021) Evolution of shares in a proof-of-stake cryptocurrency. *Management Science* 67(2):661–672.
- Saleh F (2021) Blockchain without waste: Proof-of-stake. *Review of Financial Studies* 34(3):1156–1190.
- Saleh F, Travers C (2026) An overview of Morpho Midnight. *Working Paper*.
- Shleifer A, Vishny RW (1992) Liquidation values and debt capacity: A market equilibrium approach. *Journal of Finance* 47(4):1343–1366.
- Shleifer A, Vishny RW (1997) The limits of arbitrage. *Journal of Finance* 52(1):35–55.
- Tsoukalas G, Falk BH (2020) Token-weighted crowdsourcing. *Management Science* 66(9):3843–3859.
- Wang Q (2024) Does DeFi democratize access to financial services? *Available at SSRN 4779624*.

## A Preliminaries

**Stochastic Environment** Denoting by  $\mathcal{F}_t^-$  the information available at the beginning of period  $t$  (i.e.,  $\theta_{t-1}$  together with the beginning of period innovations),  $\tilde{\theta}_t$  is  $\mathcal{F}_t^-$ -measurable whereas  $\varepsilon_t^+$  is independent of  $\mathcal{F}_t^-$  (and hence of  $\tilde{\theta}_t$ ), so that the distribution of  $\varepsilon_t^+$  conditional on  $\mathcal{F}_t^-$  is given by  $\pi_+$ . Finally, on the state space of strictly positive borrower and lender measures, the process defined by Equation (3.1) has a unique stationary distribution. We initialize the process according to this distribution, so that:

$$\{(\mu_t, \lambda_t, \psi_{t,t+1})\}_{t=1}^\infty \text{ is strictly stationary,} \quad \text{supp}(\theta_t) \subseteq [\theta_{\min}, \theta_{\max}] \subset \mathbb{R}_{++} \quad (\text{A.1})$$

where  $0 < \theta_{\min} \leq \theta_{\max} < \infty$  are constants.

**High Quality Collateral** The high quality collateral assumption states that  $F_\psi$  belongs to  $\mathcal{C}$ , the set of probability distributions  $F$  on  $\mathbb{R}$  such that  $F$  has bounded support and satisfies:

$$\int \left[ v_b(\chi(1 + \psi) + y - \bar{r}_b) - (1 - p) \max \{ v_b(\chi(1 + \psi) + y), v_b(1 + y) \} \right] dF(\psi) > p \cdot v_b(1 + y) \quad (\text{A.2})$$

$$\chi(1 + \inf \text{supp}(F)) \geq 1 \quad (\text{A.3})$$

Note that Equation (A.3) implies  $\chi(1 + \psi) \geq 1$  for all  $\psi \in [\underline{\psi}, \bar{\psi}]$ , so that  $v_b$  being strictly increasing further implies:

$$\max \{v_b(\chi(1 + \psi) + y), v_b(1 + y)\} = v_b(\chi(1 + \psi) + y) \quad \text{for all } \psi \in [\underline{\psi}, \bar{\psi}] \quad (\text{A.4})$$

## B Welfare Definitions

For each rate  $r \geq 0$ , the borrower and lender indifference cut-offs,  $\tilde{r}_b(r)$  and  $\tilde{r}_l(r)$ , are defined in Equations (5.5) and (5.6). A competitive equilibrium for credit market tightness  $\theta$  is a quoted rate  $r^{CE}(\theta) \geq 0$  satisfying:

$$1 = \theta \cdot \frac{1 - F_b(\tilde{r}_b(r^{CE}(\theta)))}{F_l(\tilde{r}_l(r^{CE}(\theta)))} \quad (\text{B.1})$$

The corresponding borrower and lender cut-offs are  $r_b^{CE}(\theta) := \tilde{r}_b(r^{CE}(\theta))$  and  $r_l^{CE}(\theta) := \tilde{r}_l(r^{CE}(\theta))$ . Existence and uniqueness of  $r^{CE}(\theta)$  are established in Lemma EC.9.

For any  $(\mu_t, \lambda_t, \varepsilon_t^+, \psi_{t,t+1})$ , equilibrium utilization  $U^* = U^*(\tilde{\theta}_t, \varepsilon_t^+)$ , borrower rate  $b^* = \rho(U^*)$ , and cut-offs  $(r_b^*, r_l^*)$ , DeFi welfare is defined as follows:

$$\begin{aligned} \mathcal{W}_t^{DeFi}(\rho; \mu_t, \lambda_t, \varepsilon_t^+, \psi_{t,t+1}) &:= \mu_t \int \mathbf{1}_{[r_b^*, \bar{r}_b]}(r) \max\{v_b(\chi(1 + \psi_{t,t+1}) + y - b^*), v_b(1 + y)\} dF_b(r) \\ &\quad + \mu_t \int \mathbf{1}_{[\underline{r}_b, r_b^*]}(r) v_b(\chi(1 + \psi_{t,t+1}) + y - r) dF_b(r) \\ &\quad + \lambda_t \int_0^{r_l^*} v_l(1 + l^*) dF_l(r) + \lambda_t \int_{r_l^*}^{\infty} v_l(1 + r) dF_l(r) \end{aligned} \quad (\text{B.2})$$

Competitive welfare  $\mathcal{W}_t^{CE}$  is defined analogously with  $U = 1$ ,  $b^* = r^{CE}(\theta_t)$ ,  $r_b^* = r_b^{CE}(\theta_t)$ ,  $r_l^* = r_l^{CE}(\theta_t)$ , and  $l^* = \min\{1 + r^{CE}, \chi(1 + \psi_{t,t+1})\} - 1$ . When  $\varepsilon_t^+$  is deterministic it is suppressed from the argument list.