Internet Appendix for "The Central Bank's Balance Sheet and Treasury Market Disruptions"

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This Internet Appendix develops a rational-expectations extension in which the supply of reserves and Treasuries follows a Markov regime process that agents internalize, and shows that doing so leaves the model's basis predictions essentially unchanged.

IA.1 Rational-Expectations Regime Extension

Setup. In the main text the model is estimated under the assumption that the supply pair $(\underline{m}_t, b_t^\tau)$ is constant at a steady-state level, while the empirical exercise feeds in the observed time variation. Since reserves market clearing implies $m_t = \underline{m}_t$ at all dates, we drop the underline and write m_t for reserve supply throughout. This appendix instead treats (m_t, b_t^τ) as a finite-state Markov process: agents know their current regime, know the transition probabilities to every other regime, and price the risk of switching into today's portfolio choices.

We discretize Treasury supply b_t^τ into $K = 10$ regimes by K-means over the January 2017–February 2025 estimation sample. We cluster on Treasury supply because it is the dominant driver of the basis—a larger float raises the dealer intermediation cost ν—and because it rises near-monotonically, so the regimes form a *supply ladder*: we order them ascending in b^τ, and the supply state moves slowly through the ladder over time. Each regime j is summarized by its mean supply $(\bar{m}_j, \bar{b}_j^\tau)$, with $\bar{m}_j$ the within-regime average reserve level, and by the empirical daily transition matrix Q. Table IA.1 reports the classification.

Regime dynamics. Within a regime the economy behaves exactly as in the main text: from the steady state α^s, a flight-to-deposit shock (rate λ) draws an α-shock α', and a reversion shock (rate λ') returns the economy to α^s. Regime switches occur at the steady state: an independent Poisson shock (rate μ_j) moves the economy from (α^s, j) to a new steady state (α^s, j') with probability $q_{jj'}$, changing the supply $(\bar{m}_j, \bar{b}^\tau_j) \mapsto (\bar{m}_{j'}, \bar{b}^\tau_{j'})$ but leaving α unchanged.

The friction. The extension turns on one assumption about the transaction cost ν. In the main text ν is the cost of intermediating Treasuries in the *secondary* market: it prices the fire-sale round trip, in which the shadow bank sells Treasuries to the traditional bank under an α-shock and buys them back at α^s. A regime switch is a different event—a change in the *supply* of Treasuries—which we take to be absorbed in the *primary* market, frictionlessly, in the amount by which the float changes, $\Delta b^\tau = \bar{b}^\tau_{j'} - \bar{b}^\tau_j$. We adopt this for three reasons. First, it preserves the tractability of the framework: it leaves the within-regime marginal values untouched, so each regime is solved exactly as in the main text. Second, it keeps the friction where the basis lives—in the secondary market for liquidity—rather than layering on a supply-side friction that the data give no reason to add. Third, it matches how Treasury supply is placed in practice: new issuance is taken down through regular, competitive auctions by primary dealers (Garbade and Ingber, 2005), and securities clear at auction at a small concession to buyers rather than at the secondary-market cost ν—if anything, buyers are paid to absorb it (Herb, 2025). The restriction to Δb^τ also keeps the equilibrium well-defined—an unbounded costless channel would let the shadow bank, which values Treasuries at ν above their price, buy without limit and compete the 2ν gap to zero—while any rebalancing beyond the new issuance still pays ν.

Per-regime equilibrium. Two consequences follow. First, the marginal value of a Treasury at (α^s, j) is single-valued, as a Markov equilibrium requires: the costless supply transition imposes no transaction-cost wedge, so the marginal value is pinned by the within-regime α-dynamics alone. Lemma A1 then applies regime by regime—the fire-sale round trip gives $\theta_j(\alpha^s) = -\nu_j$ and $\overline{\theta}_j(\alpha^s) = +\nu_j$, so the shadow–traditional marginal-value gap is $2\nu_j$, exactly as in a single regime. Second, the equilibrium is solved regime by regime: at the supply $(\bar{m}_j, \bar{b}^\tau_j)$, market clearing determines the bank Treasury positions and the steady-state triparty–deposit spread $\Theta^s_j \equiv r^{pt}(\alpha^s) - r^d(\alpha^s)$, with the marginal values held at the α-cycle boundaries. The spread Θ^s_j solves the steady-state condition stated next, and the basis is $-\Theta^s_j/2$.

Internalizing regime switching. Internalizing the switching alters only the bank value functions, through one extra Poisson jump; we derive the spread condition from them. The main-text value functions, now indexed by regime, stay linear in Treasury holdings,

$$V_j(b; \alpha) = \xi_j(\alpha) + \theta_j(\alpha)\, b, \qquad \overline{V}_j(\bar{b}; \alpha) = \overline{\xi}_j(\alpha) + \overline{\theta}_j(\alpha)\, \bar{b},$$

so the marginal values are $V_{j,b} = \theta_j$ and $\overline{V}_{j,b} = \overline{\theta}_j$. At the steady state of regime j the shadow bank faces the flight shock at rate λ (drawing $\alpha' \sim f$) and the regime shock at rate $\mu_j = \sum_{j' \neq j} q_{jj'}$ (moving to (α^s, j') at rate $q_{jj'}$). A regime switch changes the supply of Treasuries, which the bank absorbs at no transaction cost; its continuation value is then $\overline{V}_{j'}(\bar{b}; \alpha^s)$, with nothing deducted for the change in holding, and—the value function being linear in the holding—the marginal value carried into regime j' is $\overline{\theta}_{j'}(\alpha^s)$, whatever the level. The regime-j value function therefore solves

$$\rho \overline{V}_j(\bar{b}; \alpha^s) = \overline{\pi}_j(\bar{b}) + \lambda \int \left[\overline{V}_j(\bar{b}; \alpha') - \overline{V}_j(\bar{b}; \alpha^s)\right] f(\alpha')\, d\alpha' + \sum_{j' \neq j} q_{jj'} \left[\overline{V}_{j'}(\bar{b}; \alpha^s) - \overline{V}_j(\bar{b}; \alpha^s)\right],$$

with $\overline{\pi}_j$ the flow payoff, whose derivative $\partial \overline{\pi}_j / \partial \bar{b} = r^\delta_j(\alpha^s) - r^p_j(\alpha^s)$ is the cash–futures spread. Differentiating in $\bar{b}$ (using $\overline{\theta}_j(\alpha') = -\nu_j$ in the fire-sale region $\mathcal{F}_j$), and repeating for the traditional bank (marginal flow $r^\delta_j - r^d_j - \phi \vartheta^e_j$, with $\theta_j(\alpha') = +\nu_j$ in $\mathcal{F}_j$), gives the two steady-state envelopes

$$(\rho + \lambda + \mu_j)\, \overline{\theta}_j(\alpha^s) = r^\delta_j(\alpha^s) - r^p_j(\alpha^s) + \lambda \int \overline{\theta}_j(\alpha')\, f(\alpha')\, d\alpha' + \sum_{j' \neq j} q_{jj'}\, \overline{\theta}_{j'}(\alpha^s), \tag{IA.1}$$

$$(\rho + \lambda + \mu_j)\, \theta_j(\alpha^s) = r^\delta_j(\alpha^s) - r^d_j(\alpha^s) - \phi \vartheta^e_j(\alpha^s) + \lambda \int \theta_j(\alpha')\, f(\alpha')\, d\alpha' + \sum_{j' \neq j} q_{jj'}\, \theta_{j'}(\alpha^s). \tag{IA.2}$$

The last term in each is the only new object: at a switch the carried bond is worth the destination marginal value $\overline{\theta}_{j'}(\alpha^s)$, resp. $\theta_{j'}(\alpha^s)$—continuous, with no $\pm\nu$ transaction-cost wedge, exactly because the supply change is frictionless. Subtract (IA.2) from (IA.1); using $\overline{\theta}_j(\alpha^s) - \theta_j(\alpha^s) = 2\nu_j$ (Lemma A1), $r^p_j - r^{pt}_j = \phi \vartheta^e_j$, $\Theta^s_j \equiv r^{pt}_j(\alpha^s) - r^d_j(\alpha^s)$, and $\nu_j = \nu_0 (\bar{b}^\tau_j / \bar{b}^\tau)^{\gamma_\nu}$, this rearranges to

$$\Theta^s_j + 2(\rho + \lambda + \mu_j)\, \nu_j + 2\lambda \int_{\alpha' \in \mathcal{F}_j} \nu_j\, f(\alpha')\, d\alpha' \;-\; \lambda \int_{\alpha' \notin \mathcal{F}_j} \left(\overline{\theta}_j(\alpha') - \theta_j(\alpha')\right) f(\alpha')\, d\alpha' \;=\; \sum_{j' \neq j} q_{jj'}\, 2\nu_{j'}. \tag{IA.3}$$

The right-hand side is the regime-switch continuation of the gap, $\sum_{j' \neq j} q_{jj'} \left[\overline{\theta}_{j'}(\alpha^s) - \theta_{j'}(\alpha^s)\right] = \sum_{j' \neq j} q_{jj'}\, 2\nu_{j'}$, the destination gap being $2\nu_{j'}$ by Lemma A1 in regime j'. The left-hand side is the main text's constant-supply condition with the discount $\rho + \lambda$ raised to $\rho + \lambda + \mu_j$; setting $\mu_j = 0$, so both new pieces vanish, recovers it. The condition is solved exactly, one regime at a time (its right-hand side is fixed by supply). Writing $\Theta^{s,0}_j$ for the solution at $\mu_j = 0$—the constant-supply spread—the change in the basis from internalizing switching is $\delta_j \equiv \left(-\tfrac{1}{2}\Theta^s_j\right) - \left(-\tfrac{1}{2}\Theta^{s,0}_j\right) = \tfrac{1}{2}\left(\Theta^{s,0}_j - \Theta^s_j\right)$, read off the exact per-regime solutions and reported in Table IA.3. It also admits a transparent leading-order approximation: subtracting the $\mu_j = 0$ condition from (IA.3), the fire-sale integrals and the $2(\rho+\lambda)\nu_j$ term enter both and approximately cancel—their dependence on Θ^s_j is higher order—leaving $\Theta^s_j - \Theta^{s,0}_j \approx \sum_{j' \neq j} q_{jj'} 2\nu_{j'} - 2\mu_j \nu_j$, so that

$$\delta_j \approx \widetilde{\delta}_j \equiv \sum_{j' \neq j} q_{jj'} \, (\nu_j - \nu_{j'}),$$

the switch-rate–weighted change in the transaction cost across regimes. This approximation $\widetilde{\delta}_j$ captures the sign and structure of the correction but, by dropping the $O(\rho+\lambda)$ discount factor, overstates the magnitude of the exact δ_j by roughly an order of magnitude. Because δ_j is the *change* in ν across a switch (not its level), weighted by the switch intensity $q_{jj'}$, and adjacent regimes differ in ν only marginally while being highly persistent ($q_{jj'}$ of order one per year), $|\delta_j| \leq 5.2 \times 10^{-5}$ basis points (Table IA.3)—negligible even though $\nu(b^\tau)$ sets the cross-regime basis *level* itself. The calibrated and estimated parameters are those of the main text (Table 1).

Solution algorithm.

1. *Classify.* Cluster the daily Treasury supply $\{b^\tau_t\}$ into K regimes by K-means; order them ascending in mean supply and set $(\bar{m}_j, \bar{b}^\tau_j)$ to the within-regime means.
2. *Transition matrix.* Build Q from the observed regime sequence, restricted to adjacent transitions (tridiagonal); the exit rate is $\mu_j = \sum_{j' \neq j} q_{jj'}$.
3. *Per-regime basis.* For each j, solve (IA.3) for Θ^s_j (the right-hand side is fixed by supply, so the regimes are independent); the basis is $-\Theta^s_j/2$, with $\nu_j = \nu_0(\bar{b}^\tau_j/\bar{b}^\tau)^{\gamma_\nu}$.
4. *Map to dates.* Assign each date t to its regime $j(t)$; the model basis is $-\Theta^s_{j(t)}/2$, censored to the structural floor $\rho\,\nu_{j(t)}$ in the 2020–2021 supplementary-leverage-ratio (SLR) relief window, as in the main text.

Empirical regime classification. Table IA.1 reports each regime's mean supply $(\bar{m}_j, \bar{b}^\tau_j)$, its daily persistence, and its mean duration. The regimes run from the low-supply configuration of 2017–2018 (regime 1) to the high-supply quantitative-tightening configuration of 2023–2025 (regime 10). They are persistent—daily persistence between 0.98 and 1.00, mean durations from about two months to a year—so the supply state turns over slowly.

Transition matrix. Table IA.2 reports the estimated daily transition matrix Q, restricted to adjacent transitions (tridiagonal). Each regime stays put with probability 0.98–1.00 and

Table IA.1: Empirical regime classification (K-means, $K = 10$, on Treasury supply b^{τ}, ordered ascending, 2017–2025). Reserves $\bar{m}$ and Treasury supply $\bar{\tau}^h$ are in trillions of dollars; mean duration in days.

Regime	$\bar{m}$	$\bar{\tau}^h$	Persistence	Mean duration
1	2.21	18.42	0.997	371.0
2	1.69	19.58	0.997	309.0
3	2.04	21.02	0.994	175.0
4	2.93	22.08	0.993	136.0
5	3.76	23.25	0.991	112.0
6	4.27	24.59	0.992	130.0
7	3.60	26.33	0.992	130.0
8	3.22	27.49	0.990	105.0
9	3.46	28.53	0.982	55.2
10	3.37	29.70	0.992	127.0

otherwise moves one rung—almost always up, since Treasury supply rises near-monotonically, and occasionally down in the high-supply regimes. The supply thus drifts gradually through neighboring regimes rather than jumping, mirroring the slow rise of Treasury supply in Figure 1.

Quantitative result. Table IA.3 reports the per-regime basis $-\Theta^s_j/2$ and its change δ_j from the constant-supply value. The basis ranges from near zero to 37 basis points across the supply ladder, while δ_j is at most 5.2×10^{-5} basis points—four orders of magnitude below the ~ 0.5 basis-point precision of the 90-day moving-average data target. Internalizing rational-expectations regime switching is therefore quantitatively irrelevant, exactly as the cancellation in (IA.3) shows: the per-regime steady states are the equilibrium. Figure 1 plots the model basis against the observed series; the regime step tracks the level of the basis as Treasury supply rises, reaching the stress band in the high-supply 2023–2025 regime.

Discussion. The effect is negligible for two compounding reasons, both visible in (IA.3). With mean regime durations of months to a year, the exit rate μ_j is small, so agents heavily discount future switches; and the two terms that switching adds—a faster discount and a destination continuation—nearly cancel, because the marginal-value gap $2\nu_j$ moves little across adjacent supply regimes. The conclusion is robust to the number of regimes K: it rests only on the slow, near-monotone drift of Treasury supply, which any reasonable discretization preserves.

Table IA.2: Estimated daily transition matrix Q ($K = 10$): entry (j, j') is the probability of moving from regime j today to j' tomorrow. Q is tridiagonal: supply moves only to an adjacent regime.

Regime	1	2	3	4	5	6	7	8	9	10
1	0.997	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.997	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.994	0.006	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.993	0.007	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.991	0.009	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.992	0.008	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.992	0.008	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.990	0.010	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.982	0.009
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.992

Table IA.3: Per-regime basis $-\Theta^s_j/2$ and its change δ_j from the constant-supply value ($\mu_j = 0$), in basis points.

Regime	Basis	δ_j
1	10.097	-7.8×10^{-6}
2	20.023	-1.3×10^{-5}
3	20.749	-1.8×10^{-5}
4	11.559	-2.8×10^{-5}
5	2.327	-4.3×10^{-5}
6	0.031	-5.2×10^{-5}
7	15.823	-3.7×10^{-5}
8	28.651	-4.4×10^{-5}
9	28.894	-8.2×10^{-6}
10	37.207	4.4×10^{-5}

Replication notes. The figure and tables are reproduced from `code/matlab/re_regimes_v7/` in the replication package:

```
matlab -batch "make_regimes"    % Tables IA.1-IA.3 + Figure regimes.pdf
```

References

d'Avernas, A., B. Han, and Q. Vandeweyer (2025). Intraday liquidity and money market dislocations. *Management Science*.

Garbade, K. D. and J. F. Ingber (2005). The Treasury auction process: Objectives, structure, and recent adaptations. *Federal Reserve Bank of New York Current Issues in Economics and Finance* 11(2).

Herb, P. (2025). The Treasury auction risk premium. *Journal of Banking & Finance* 170.

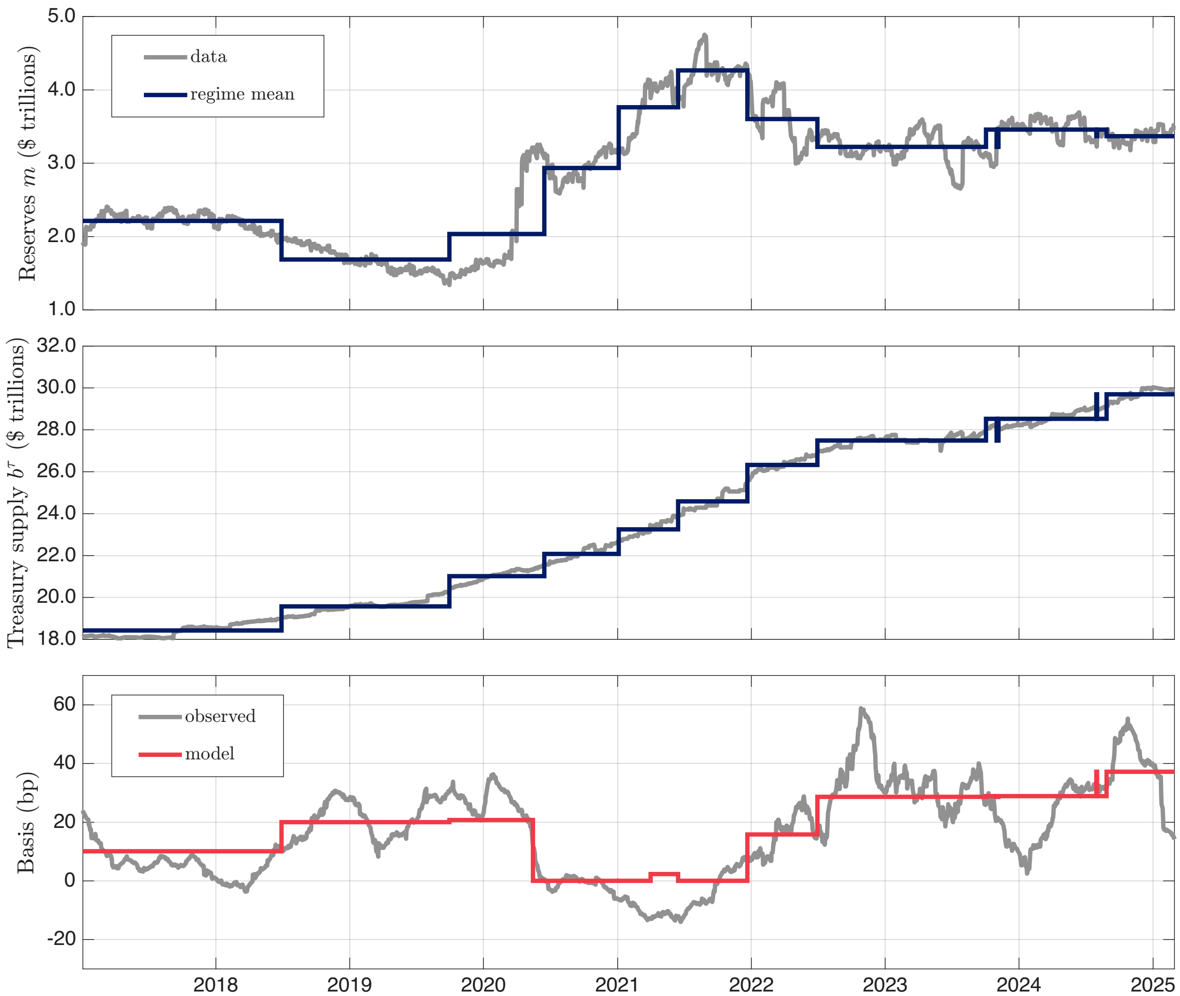

Figure 1: The supply-regime classification and the model basis. *Top:* reserves m_t; *middle:* Treasury supply b_t^τ; each with the regime-mean step (blue) over the daily series (grey), showing which of the ten regimes each date is assigned to. *Bottom:* the observed cash–futures basis (grey) and the per-regime model basis (red). The model step tracks the level of the basis as Treasury supply rises, reaching the stress band in the high-supply 2023–2025 regime; it stays below the short-lived moving-average peaks, which lie above the model's steady-state basis at any observed supply. In the 2020–2021 SLR-relief window the model basis is censored to its structural floor, as in the main-text estimation.