The Central Bank's Balance Sheet and Treasury Market Disruptions*

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Abstract

We study how banking regulation and the central bank balance sheet jointly influence Treasury market fragility. In a dynamic general equilibrium model, leverage-ratio requirements push Treasuries from banks to unregulated hedge funds that arbitrage the cash-futures basis, while intraday reserve regulations constrain banks' repo lending to hedge funds when repo funding is stressed. As a result, the likelihood and severity of disruptions—and, via hedge funds' compensation for liquidity risk, the cash-futures basis—depend on the size of the central bank's balance sheet and the expected duration of shocks.

Keywords: Repo, Liquidity Risk, Cash-futures, Basis Trade, Hedge Funds, Reserves

JEL Classifications: E43, E44, E52, G12

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Conflict-of-interest disclosure statement

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I have nothing to disclose.

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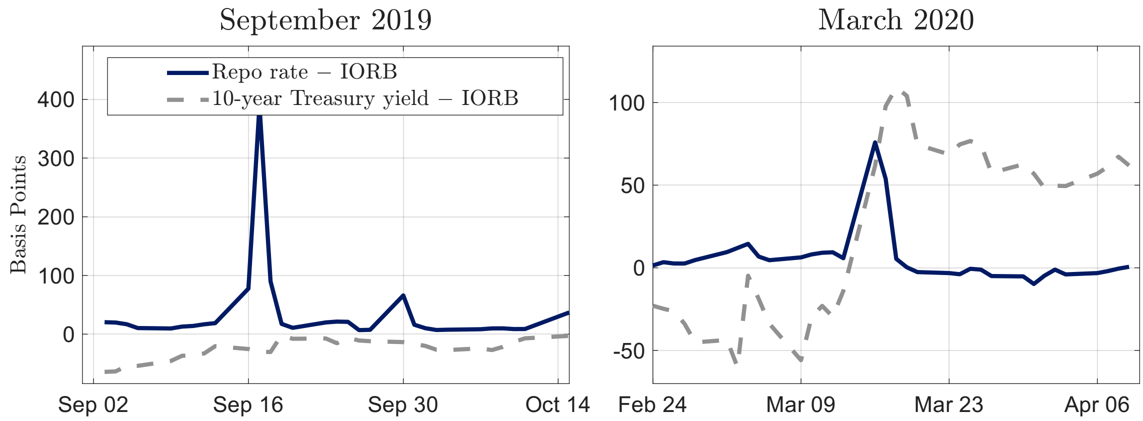


Figure 1: Money market and Treasury rates around the September 2019 and March 2020 episodes. Each panel plots the spread of the Treasury general-collateral (GCF) repo rate (DTCC GCF Repo index for Treasury collateral; solid line) and of the 10-year Treasury yield (dashed line) to the interest on reserve balances (IORB; before July 2021, the interest on excess reserves), in basis points. Left panel: September–October 2019. Right panel: February–April 2020.

In recent years, advanced economies have repeatedly faced disruptions in Treasury and repo markets, with material implications for financial stability, market functioning, and the perceived safety of sovereign debt.[1] These disruptions have coincided with greater participation by leveraged non-bank financial institutions—particularly hedge funds and pooled liability-driven investment (LDI) funds—and growing discrepancies between Treasury securities prices and those of corresponding futures contracts, a phenomenon known as the *Treasury cash-futures basis.* While these developments have prompted extensive policy discussions (Duffie, Geithner, Parkinson, and Stein, 2022; Kashyap, Stein, Wallen, and Younger, 2025) and academic research (Vissing-Jorgensen, 2021; Barth and Kahn, 2025; Barth, Kahn, and Mann, 2023), the literature still lacks a comprehensive, dynamic framework to analyze the interactions among market participants, regulatory constraints, and disruption expectations.[2] Figure 1 illustrates the two U.S. episodes at the center of our analysis: in September 2019, the general-collateral repo rate spiked to nearly 400 basis points above the interest on reserve balances while Treasury yields barely moved; in March 2020, repo rates rose more moderately, but the 10-year Treasury yield surged as hedge funds liquidated Treasury positions.

In this paper, we develop a dynamic model to study how the central bank's balance sheet influences Treasury market fragility and the Treasury cash-futures basis. In the model, regulated banks interact with leveraged non-bank institutions—such as hedge funds engaged in relative-

[1]Notable events include the September 2019 repo market spike, the March 2020 Treasury sell-off in the U.S. amid the COVID-19 pandemic, and the September 2022 surge in U.K. gilt yields following the Truss government's mini-budget announcement.

[2]Besides the direct impact on Treasury markets, instability in repo rates can also propagate uncertainty into broader derivatives markets, particularly those referencing the Secured Overnight Financing Rate (SOFR) as a benchmark. Such volatility could impair pricing accuracy and reduce hedging effectiveness across numerous financial instruments.

value arbitrage and leveraged LDI funds—to determine leverage and demand for Treasuries and repo funding. Two regulatory constraints on banks jointly drive the model's central mechanism. The first is the leverage ratio. It makes bank balance-sheet space costly, and provides a comparative advantage for shadow banks to hold Treasuries in a cash-futures basis trade. Such a trade, however, exposes those shadow banks to liquidity risk arising from the instability of repo markets which can make the trade unprofitable and force costly liquidations. In competitive equilibrium, Treasury holdings and repo financing are such that the basis equals the shadow cost of banks' balance sheet plus the required compensation for liquidity risk for shadow banks. The second piece of regulation is the intraday liquidity requirement. It ties banks' repo lending to their reserve holdings and binds probabilistically when reserves are scarce and repo becomes expensive. Combined, those two constraints link the central bank's balance sheet to the basis through two channels. Ex ante, shadow banks anticipate funding stress and demand additional return compensation. Ex post, binding events generate repo rate spikes such as those of September 2019 and March 2020. The main contribution of our paper is to characterize Treasury market fragility and the Treasury cash-futures basis as equilibrium outcomes of this interplay between leverage-ratio requirements, intraday liquidity regulations, and the size of the central bank's balance sheet. Figure 2 illustrates this relationship by plotting the time series of the cash-futures basis from 2017 to early 2025, along with the fitted values from a linear regression of the basis on the quantities of Treasuries and central bank reserves outstanding—the latter directly determined by the size of the central bank's balance sheet.

Our model comprises three sectors—traditional banks, shadow banks, and households—and incorporates four key elements. Households demand liquid assets through a convex combination of bank deposits and repos;[3] regulatory leverage ratio constraints require banks to maintain equity proportional to their total assets; intraday liquidity regulations require traditional banks to pre-fund their repo lending activities with central bank reserves; and transacting Treasuries is costly.

In the model, households' demand for money-like deposits and repos creates a role for financial institutions to engage in liquidity transformation. Both traditional and shadow banks undertake this activity by holding duration-hedged Treasuries and issuing money-like assets to households. Shadow banks, however, have a comparative advantage in this activity, as they are exempt from capital requirements and can therefore avoid costly equity financing and fund their positions entirely with money-like repos. This regulatory arbitrage drives a disproportionate portion of Treasury holdings onto hedge fund balance sheets.[4]

Although shadow banks benefit from being unregulated, they face a disadvantage relative to banks, as they can issue repos but not deposits. Consequently, a sudden increase in households' preference for deposits cannot be met solely by shadow banks and must be absorbed by tradi-

[3] I.e., accessed through an unmodeled money market fund.

[4] This phenomenon corresponds to the empirical observation of Barth and Kahn (2025) of increased hedge fund involvement in Treasury and futures markets since the introduction of the supplementary leverage ratio (SLR) for large banks in the U.S. in 2014.

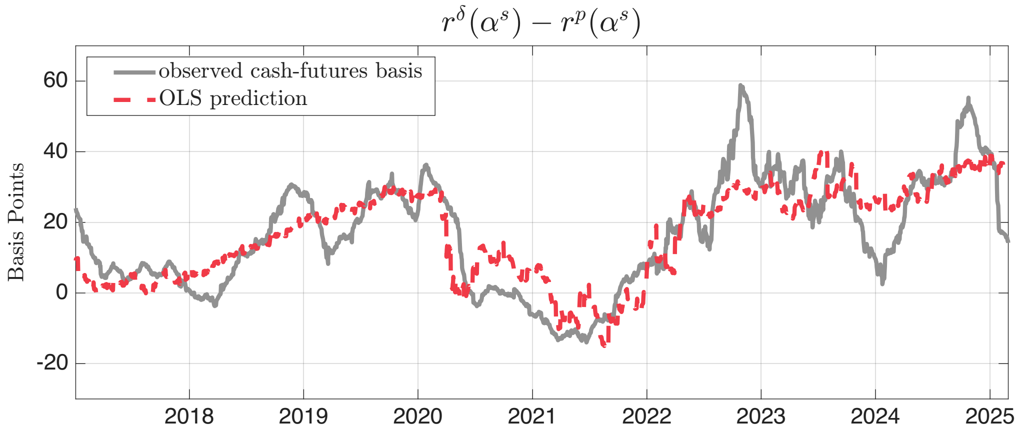


Figure 2: Observed and OLS-predicted cash-futures basis. Predictions are taken from a linear regression of the observed cash-futures basis on the quantity of Treasuries and reserves outstanding (see Appendix C for the regression Table). The observed cash-futures basis is the 5-year tenor series from Barth and Kahn (2025), smoothed with a 90-day centered moving average.

tional banks. Provided traditional banks hold sufficient reserves to satisfy their intraday liquidity requirements, they can accommodate this shift by issuing (borrowing) additional deposits and lending the proceeds via repos to shadow banks.[5] In such a scenario, repo rates remain anchored at the interest on reserves and shadow banks fully retain their Treasury holdings.

However, binding intraday requirements can significantly alter this dynamic. Under these conditions, banks' ability to supply liquidity to the repo market and absorb the preference shock is limited by the amount of reserves held at the start of the day.[6] In such cases, repo markets become effectively rationed, causing the repo rate to rise above the interest on reserves to reflect a positive wedge in households' marginal preferences.

The possibility of such spikes in repo rates has important implications for shadow banks, whose liquidity transformation activities involve borrowing in repo markets to finance illiquid Treasury positions (i.e., subject to transaction costs). Namely, when the repo rate spikes above the interest on reserves, shadow banks incur losses on these cash-futures positions, as the financing cost exceeds the yield earned on duration-hedged Treasuries. For shocks that are sufficiently large, persistent, or both, the repo rate may cross a threshold beyond which it becomes more economical to incur transaction costs and liquidate the Treasury portfolio altogether. Beyond this threshold, stresses in funding repo markets spread to Treasury markets.

When making portfolio decisions, shadow banks account for the liquidity risk they are taking when financing Treasury holdings through potentially volatile repo markets. Anticipating this

[5]This practice, in which banks lend into recurring distortions in funding markets, has been documented by practitioners and is commonly described as "lending-of-next-to-last-resort" (Pozsar, 2019) or "reserves-draining intermediation" (Correa, Du, and Liao, 2020).

[6]Under Basel III regulations, implemented in the U.S. via Regulation YY (12 CFR Part 252), banks must prefund daily reserve outflows and are effectively restricted from using central bank intraday overdrafts (see d'Avernas, Han, and Vandeweyer (2025) for details).

risk, shadow banks scale down their provision of liquidity transformation relative to a scenario with reduced liquidity concerns. As a result, the equilibrium cash-futures basis—the return on repo-financed and duration-hedged Treasuries—incorporates shadow banks' required compensation for bearing this liquidity risk.

A direct implication of this insight is that the size and composition of the central bank's balance sheet play a crucial role in determining Treasury market fragility and thus the equilibrium level of the cash-futures basis. This occurs because the asset and liability sides of the balance sheet each interact distinctly with regulatory constraints. On the asset side, reduced central bank Treasury holdings tighten traditional banks' leverage ratio constraints and increase shadow banks' demand for repo financing, thereby raising the likelihood of both repo rate spikes and Treasury sales. On the liability side, fewer reserves limit banks' ability to expand repo lending during repo supply shocks, further amplifying disruption risks.[7]

The model also provides a rationale for the observation that the March 2020 COVID-19 shock triggered significant Treasury fire sales, whereas earlier episodes, such as the September 2019 event, primarily caused repo rate spikes with limited Treasury liquidations. In our framework, trading costs create an inaction region, implying that the relative market response depends crucially on agents' expectations about shock duration. Specifically, short-lived shocks tend to raise repo rates but do not trigger Treasury sales, as shadow banks prefer temporarily paying elevated repo rates over incurring fixed transaction costs from Treasury liquidations. Conversely, if a shock is expected to persist, shadow banks optimally pay these costs upfront and sell their Treasuries, avoiding sustained high repo funding rates. This distinction aligns with observed market behavior: the September 2019 shock was widely viewed as transient, whereas the March 2020 "dash-for-cash" toward deposits was likely anticipated to last longer. Quantitatively, our model indicates that rationalizing hedge funds' Treasury liquidations in March 2020 requires an increase in the risk-neutral expected shock duration from 1 day to roughly 48 days.

Expectations about shock frequency also shape Treasury-market fragility. When shadow banks anticipate less frequent shocks, they expand repo-financed Treasury holdings, which in turn raises the intensity of shocks conditional on arrival. This substitution from frequency to intensity concentrates liquidity risk in specific states of the world, generating a Treasury "volatility paradox" akin to Brunnermeier and Sannikov (2014b). This result underscores the importance of a dynamic framework for evaluating policy counterfactuals, as shadow banks continuously adapt their balance-sheet decisions to prevailing risk conditions.

On the normative side, the model yields a concrete reserve-supply rule that eliminates repo rate spikes: maintaining reserves above a fixed fraction of the outstanding Treasury supply—with the fraction pinned down by the regulatory intraday-liquidity parameter—keeps the intraday liquidity constraint slack in every state, with departures from this benchmark absorbed by

[7]Empirical evidence from March 2020 supports this mechanism: the Federal Reserve's Treasury purchases, funded by reserve expansion, significantly reduced stress in repo and Treasury markets (Vissing-Jorgensen, 2021).

selective open-market operations and the Standing Repo Facility. We also discuss how this rule, by tying the central bank's balance sheet to Treasury supply, has implications for the broader monetary–fiscal interaction—in particular, the risk that a sustained accommodative balance-sheet stance opens a channel toward fiscal dominance.

Finally, we structurally estimate our model and conduct a series of quantitative counterfactual exercises. The empirical fit provides a validation of the model's central mechanism: fluctuations in the Treasury cash-futures basis arise largely from the size of the central bank's balance sheet vis-à-vis the outstanding supply of Treasuries. The estimated parameters also yield economically plausible and internally consistent values, indicating that the economic mechanism of our framework can account for the observed Treasury cash-futures basis.

In the first counterfactual exercise, we simulate the time series of the cash-futures basis under a counterfactual scenario in which the level of reserves remains fixed at its mid-September 2019 level of roughly \$1.4 trillion—its lowest up to that point in the sample—just before the Federal Reserve resumed open market operations to inject additional liquidity into the system. Under this scenario, the cash-futures basis would have been substantially higher throughout the post-2020 period—hovering around 30 basis points through 2020 and climbing to roughly 55 basis points by the end of 2021, when the observed basis was at or below zero—and would have peaked near 90 basis points in late 2024. In a second exercise, we consider a scenario in which the U.S. Treasury maintains a constant level of outstanding debt throughout the sample period, starting in January 2017. In this case, the cash-futures basis would be substantially lower, remaining close to zero for most of the post-2020 sample, as actual reserves are sufficiently ample to sustain this lower Treasury supply without significant repo spike risk.

Related Literature Our paper contributes to a growing literature investigating how financial frictions and regulatory innovations lead to persistent pricing deviations, regulatory arbitrage, and fragility in the Treasury and repo markets. We extend this literature by jointly analyzing the repo and Treasury markets within a dynamic equilibrium framework, yielding new insights into how regulations shape institutional portfolio decisions and pricing dynamics, with a particular focus on the central bank balance sheet. In doing so, our work aligns closely with studies highlighting intermediaries and their financial constraints as central determinants of asset prices (Brunnermeier and Sannikov, 2014a; He and Krishnamurthy, 2013; He, Kelly, and Manela, 2017). In particular, our model builds upon prior works that integrate central bank reserves and interbank market structures into general equilibrium models (Bianchi and Bigio, 2022; Piazzesi and Schneider, 2018; De Fiore, Hoerova, and Uhlig, 2018).

Studying funding markets, several studies highlight how regulatory frictions generate quarter-end anomalies in repo and FX swap markets. Specifically, Munyan (2017), Klingler and Syrstad (2021), and Du, Tepper, and Verdelhan (2018) show that foreign dealers shrink their balance sheets to comply with leverage ratio requirements, driving predictable quarter-end spikes in repo rates and cross-currency bases. Relatedly, Diamond, Jiang, and Ma (2024) find that large bank balance sheets may hinder banks' ability to lend. Boyarchenko, Eisenbach, Gupta, Shachar, and Van Tassel (2018) demonstrate how dealer balance sheet costs influence repo pricing and arbitrage funding conditions for non-bank intermediaries. Our work provides a micro-foundation for the effect of a bank leverage ratio on financial intermediation and studies the mechanism behind those empirical patterns.

Furthermore, a series of works has emphasized the importance of banks' reserves in the context of new intraday liquidity regulations as a key driver of repo rate spikes. Pozsar (2019) first raised concerns about intraday liquidity as a binding constraint for banks and potential implications for repo markets. Correa, Du, and Liao (2020) document how banks employ "reserves-draining" intermediation strategies to lend in money markets and circumvent regulatory limits during quarter-end shocks. D'Avernas, Han, and Vandeweyer (2025) highlight that under an intraday liquidity constraint, the relevant measure of excess reserves is an order of magnitude higher than under a traditional (overnight) reserve requirement regime. Copeland, Duffie, and Yang (2025) demonstrate that the quantity of reserves at large repo-dealer banks is a predictor of repo rate spikes during times of repo stresses. Yang (2022) shows how intraday liquidity constraints amplify the severity of repo rate spikes during episodes of market stress. In this work, we embed this intraday liquidity constraint in a model that also features a leverage constraint and study the interaction between the two constraints in a general equilibrium framework.

The role of various frictions in driving the September 2019 spike is further examined by Gagnon and Sack (2019); Avalos, Ehlers, and Eren (2019); Afonso, Cipriani, Copeland, Kovner, La Spada, and Martin (2020); Anbil, Anderson, and Senyuz (2021). d'Avernas and Vandeweyer (2024) and Anbil, Anderson, Cohen, and Ruprecht (2023) emphasize liquidity demand from non-bank financial institutions and the floor role of reverse repo facilities for triparty repo rates, while Ma, Eisenschmidt, and Zhang (2024) and Huber (2023) analyze the role of imperfect competition in repo spreads and its implications for monetary policy.

Further, He, Nagel, and Song (2022) directly link balance-sheet frictions to the spike in Treasury yields in March 2020 within a preferred-habitat model wherein dealers incur increased holding costs when absorbing sector-wide Treasury liquidations. In contrast, our paper explicitly investigates the conditions under which Treasury fire sales emerge endogenously as equilibrium outcomes. Specifically, we analyze how hedge funds accumulate Treasuries to exploit arbitrage opportunities in the cash-futures basis and how market disruptions are mitigated by banks acting as "lenders-of-next-to-last-resort" in the repo market. In related work, Ma, Xiao, and Zeng (2022) highlight forced liquidations by mutual funds as a key driver of market disruptions in March 2020, Eisenbach and Phelan (2025) develop a global-game model of endogenous Treasury liquidations under anticipated dealer balance-sheet constraints, and Acharya and Rajan (2024); Acharya, Chauhan, Rajan, and Steffen (2023) identify a liquidity ratchet effect among banks, whereby quantitative tightening—and the resulting reserve drain—increases banks' exposure to liquidity risk.

Our paper also relates to the literature on the shadow banking sector as a product of regulatory arbitrage (Plantin, 2015; Luck and Schempp, 2014; Huang, 2018; Moreira and Savov, 2017; Begenau and Landvoigt, 2021; d'Avernas, Vandeweyer, and Darracq-Pariès, 2023), and to Diamond (2020), who highlights the demand for money-like assets as a driver of intermediation and the role of quantitative easing in alleviating intermediation constraints.

Finally, several studies have examined arbitrage trades and associated disruptions in Treasury markets. Fleckenstein and Longstaff (2020); Barth and Kahn (2025); Barth, Kahn, and Mann (2023) document persistent positive episodes in the cash-futures basis, emphasizing the recent substantial involvement of hedge funds and the importance of asset managers' demand for duration exposure. As compared to this work, Barth and Kahn (2025); Kruttli, Monin, Petrasek, and Watugala (2025) emphasize the additional potential role of margin calls on futures contracts in driving Treasury fire-sales. In subsequent work, Kashyap, Stein, Wallen, and Younger (2025) investigate a similar setting involving hedge funds and asset managers, concluding that targeted Fed purchases of bundles containing Treasuries and futures could alleviate market dysfunction without impacting broader asset prices. On a closely related topic, Jermann (2020); Du, Hébert, and Li (2023); Hanson, Malkhozov, and Venter (2024) analyze the Treasury-swap basis, highlighting dealer balance sheet constraints and convergence risks associated with these trades.

Our paper is organized as follows: Section 1 presents our dynamic asset pricing model with financial frictions. Section 2 derives the main theoretical results. Section 3 contains the quantitative analysis. Section 4 concludes.

1 Model

Let $(\Omega, \mathcal{F}, (\mathcal{F}_t)_{t\geq 0}, \mathbb{P})$ be a filtered probability space satisfying the usual conditions. All stochastic processes are adapted to $(\mathcal{F}_t)$. Time is continuous, $t \in [0, \infty)$. The economy is populated by a representative household that owns a traditional dealer-bank and a shadow bank, as well as the Treasury and a central bank. To fix ideas about sectoral portfolio holdings in the model, Figure 3 depicts an instance of the balance sheets of the different sectors in the economy. The Treasury issues Treasury bonds against households' future tax liabilities; the central bank holds Treasury bonds and issues reserves to the banking sector; households invest their wealth and future tax liabilities in deposits, equity, and repos through dealers. Traditional banks hold reserves and lend in repo, funded by issuing deposits and equity; more generally, they can also hold Treasury bonds and either lend or borrow in repo. Shadow banks hold Treasury bonds funded with repo. In the model, positions are the result of endogenous portfolio optimization under institutional constraints.

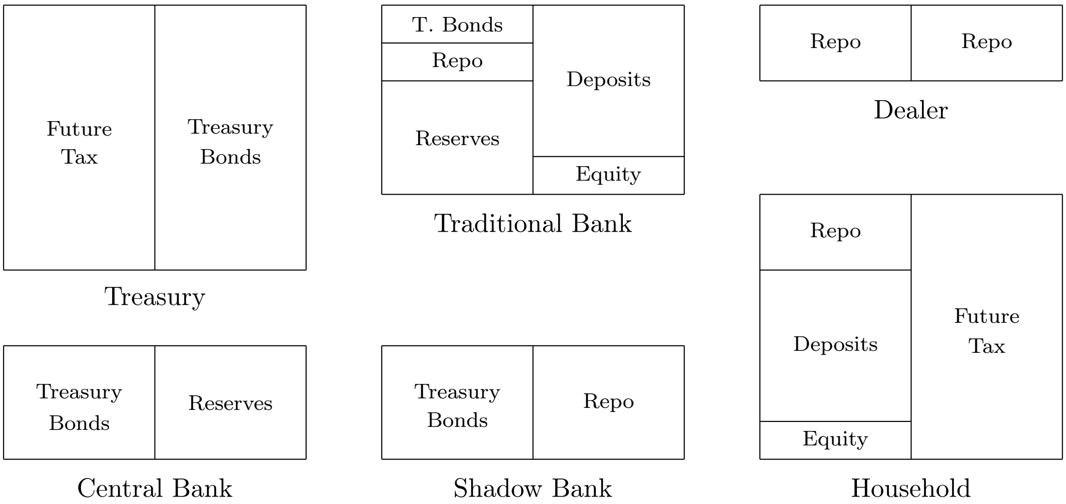


Figure 3: Chart of Sectors' Balance Sheets

1.1 Environment

Preferences Households have risk-neutral preferences over their consumption c_t^h with a time preference ρ and value liquidity services provided by holding repos and deposits:

$$\mathbb{E}_t \left[\int_t^\infty e^{-\rho(u-t)} \left(c_u^h + \beta \log(h(d_u^h, p_u^h; \alpha_u)) \right) du \right], \tag{1}$$

where h is a Cobb-Douglas liquidity services aggregator between deposits and repos,

$$h(d_t^h, p_t^h; \alpha_t) = (d_t^h)^{\alpha_t} (p_t^h)^{1-\alpha_t}. \tag{2}$$

The time-varying parameter $0 < \alpha_t < 1$ corresponds to the preference of households for holding deposits relative to repos (the deposit share).

Financial Assets Every financial asset held by an institution in the economy corresponds to a liability of another institution, and all financial institutions are ultimately owned by households. Because households are risk-neutral, the benchmark risk-free rate of the economy, the return on equity r_t^e, is equal to the households' time preference rate ρ in equilibrium.

Treasury Bonds and Futures We denote the spot price of zero-coupon Treasury bonds P_t. The price follows the process: $dP_t = P_t(r_t^b dt + \sigma_t^b dZ_t)$, where $\{Z_t\}$ is a standard Brownian motion representing an interest rate risk factor. Treasury futures contracts mature over an infinitesimal horizon dt, with the futures price given by $P_t(1 + r_t^\delta dt)$, where r_t^δ denotes the cash-futures implied rate and $r_t^\delta - r_t^p$ is the *Treasury cash-futures basis*—that is, the excess return from an arbitrage strategy that holds a Treasury bond, finances it in the repo market, and hedges its duration risk by shorting a Treasury futures contract.

To align with the paper's focus on the cash-futures basis trade and its associated liquidity risk, we assume that Treasury holdings are always paired with an offsetting short futures position that is sold to households. Consequently, households' futures positions precisely offset the duration risk arising from their tax liabilities.[8]

Treasury The Treasury issues bonds against the future tax liabilities of households and is responsible for administering redistributive lump-sum tax policies.

$$\tau_t^h = b_t^\tau, \tag{3}$$

where b_t^τ is the quantity of bonds issued by the Treasury, and τ_t^h is the present value of households' future tax liability.

Central Bank The central bank holds Treasury bonds, $\underline{b}_t$, financed by reserves held by banks $\underline{m}_t$. The underlined notation denotes central bank's holdings. Thus, the balance sheet constraint for the central bank is given by

$$\underline{b}_t = \underline{m}_t. \tag{4}$$

For simplicity, we assume that the central bank always operates with zero net worth and instantaneously transfers all seigniorage revenues to the Treasury. The budget constraint for the Treasury is therefore given by

$$d\tau_t = (b_t^\tau - \underline{b}_t)\frac{dP_t}{P_t} + \underline{m}_t r_t^m dt. \tag{5}$$

To offset any fluctuation in Treasury bond value and pay interests, the Treasury collects taxes $d\tau_t$ from households and seigniorage revenues rebated from the central bank.[9] While both the exposition below and our quantitative analysis treat the supplies of reserves and Treasuries as constant, Internet Appendix IA.1 develops a rational-expectations extension in which agents internalize the empirical dynamics of reserves and Treasury supply $(\underline{m}_t, b_t^\tau)$ as a finite-state Markov process, and shows that, given the high persistence of supply regimes in the data, internalizing these dynamics has a negligible effect on equilibrium prices—justifying the constant-supply treatment we maintain.

[8]This assumption is without loss of generality in our framework, as the representative household ultimately owns all financial and government institutions. Hence, households bear the aggregate duration exposure, which exactly offsets the duration of their tax liabilities—an implication of Ricardian neutrality.

[9]Thus, our analysis assumes a monetary dominance regime, where all bond issuances are accommodated by future tax adjustments, keeping the price level constant. We therefore abstract from additional channels linking liquidity risk to the price level. The net present value of future tax liabilities must equal the outstanding amount of Treasuries.

Dealers and Repo Markets The repo market is assumed to be fully intermediated so that households exclusively invest in repos through dealers rather than directly with traditional or shadow banks.[10] Dealers are assumed to be part of a traditional bank, which means that their intermediation positions are part of traditional banks' balance sheets. We denote the rate at which dealers borrow from households as the triparty repo rate r_t^{pt} and the rate at which they lend to shadow banks as the bilateral repo rate r_t^p, which yields the intermediation spread $r_t^p - r_t^{pt}$.[11]

Shocks The relative liquidity preference parameter α_t is subject to aggregate shocks and follows

$$d\alpha_t = \begin{cases} (\alpha^s - \alpha_t)dN_t & \text{if} \quad \alpha_t \neq \alpha^s, \\ (\alpha' - \alpha_t)dN_t & \text{if} \quad \alpha_t = \alpha^s, \end{cases} \tag{6}$$

where dN_t is a Poisson process with intensity $\lambda_t > 0$ and α' is a random variable independently and identically distributed according to a uniform distribution on $(\underline{\alpha}, 1)$, where $\alpha^s < \underline{\alpha}$.

As indicated in equation (6), the economy features a steady state α^s. Upon the arrival of a Poisson shock in state $\alpha_t = \alpha^s$, the state α_t takes on a new random value α'. Following this "flight-to-deposit" shock, α increases, and households seek to reduce their repo holdings relative to deposits. Upon the arrival of a Poisson shock in state $\alpha_t \neq \alpha^s$, α_t reverts to α^s. The Poisson intensity λ_t is equal to λ if $\alpha_t = \alpha^s$ and equal to λ' otherwise. Hence, λ governs the arrival rate of an aggregate shock while $1/\lambda'$ determines the expected duration of the shock.

Banking Regulation Traditional banks operate under two regulatory constraints that do not apply to shadow banking institutions. First, traditional banks have to abide by a (non-risk-weighted) leverage regulation stipulating that the value of their equity e_t should be greater than a fraction ϕ of their liabilities ℓ_t. Second, traditional banks face an intraday liquidity constraint, which prevents them from lending more in repos p_t than a multiple κ of their central bank reserve holdings m_t, where κ captures the intraday recirculation of reserves and can exceed one. We discuss these regulatory constraints further in Section 1.3.

[10]This assumption aligns with the actual institutional framework in the U.S., where the vast majority of repo transactions are effectively intermediated by securities dealers. For further institutional details on repo markets, we refer to the work of Copeland, Martin, and Walker (2014).

[11]We adopt these terminologies to match the U.S. institutional practice whereby money market funds (here abstracted from) lend in triparty repo, in which a third-party clearing bank handles the management of the collateral. In contrast, hedge funds tend to borrow in direct bilateral repo markets.

1.2 Agents' Problems

Traditional Banks Traditional banks face a Merton's (1969) portfolio choice problem augmented by transaction costs. Namely, they maximize their lifetime expected profits:

$$\max_{\{b_u \geq 0, m_u \geq 0, f_u, p_u, x_u \geq 0, d_u \geq 0, e_u \geq 0\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} d\pi_u \right], \tag{7}$$

subject to the profit flow equation:

$$\begin{aligned} d\pi_t = & \left(b_t r_t^b + m_t r_t^m + f_t(r_t^b - r_t^\delta) + p_t r_t^p + x_t(r_t^p - r_t^{pt}) - d_t r_t^d - e_t r_t^e \right) dt \\ & - \nu |db_t| + (b_t + f_t)\sigma_t^b dZ_t, \end{aligned} \tag{8}$$

the balance sheet constraint:[12]

$$b_t + m_t + p_t = d_t + e_t, \tag{9}$$

the intraday liquidity (IL) constraint:

$$p_t \leq \kappa m_t, \tag{10}$$

and the leverage ratio (LR) constraint:

$$e_t \geq \phi \ell_t, \tag{11}$$

Traditional banks choose Treasury bonds b_t, reserves m_t, and deposits d_t given their respective returns r_t^b, r_t^m, and r_t^d. Traditional banks also either lend or borrow in bilateral repo p_t given the interest rates in this market r_t^p. In addition, traditional banks select the size of their dealer balance sheet, x_t, and profit from the spread between the bilateral (post-intermediation) and triparty repo rates (pre-intermediation): $r_t^p - r_t^{pt}$. Finally, traditional banks purchase Treasury futures with underlying value $f_t = -b_t$ to fully hedge their duration risk exposure.

The regulatory leverage ratio defines traditional banks' liabilities as:

$$\ell_t \equiv e_t + d_t - \min\{0, p_t\} + x_t. \tag{12}$$

Through the minimum function, the portfolio weight for repos is accounted as a liability only when traditional banks are net borrowers ($p_t < 0$). As mentioned above, the dealer subsidiary's balance sheet, which consists of the intermediation of all repo positions, x_t, is added to the traditional bank balance sheet. The latter follows from the existing practices whereby internal repos between a bank and a dealer subsidiary are not netted for regulatory purposes. As mentioned above, the (IL) constraint (10) limits repo lending to a multiple κ of reserve holdings.

[12]Since the amounts of a dealer's borrowing in triparty repo and lending in bilateral repo are both equal to its balance sheet size x_t, they cancel out in the consolidated bank's balance sheet constraint.

Additionally, traditional banks consider the existence of transaction costs when trading Treasury bonds: $-\nu \times |b_t - b_{t\text{-}}|$, where ν is half of the total transaction cost and $t-$ is the time prior to the transaction. The other half of the transaction cost is borne by the counterparty to the trade.[13]

Shadow Banks The two main differences between traditional and shadow banks[14] are that shadow banks are unregulated and cannot issue bank deposits to households. Shadow banks maximize their expected profits:

$$\max_{\{\overline{b}_u \geq 0, \overline{f}_u, \overline{p}_u\}_{u=t}^{\infty}} \left[\int_t^{\infty} e^{-\rho(u-t)} d\overline{\pi}_u \right], \tag{13}$$

subject to the profit flow equation:

$$d\overline{\pi}_t = \left(\overline{b}_t r_t^b + \overline{f}_t (r_t^b - r_t^\delta) - \overline{p}_t r_t^p \right) dt - \nu |d\overline{b}_t| + (\overline{b}_t + \overline{f}_t) \sigma_t^b dZ_t, \tag{14}$$

and the balance sheet constraint:

$$\overline{b}_t = \overline{p}_t. \tag{15}$$

Shadow banks choose their holdings of Treasury bonds $\overline{b}_t$ and repo financing $\overline{p}_t$, given the respective returns r_t^b and r_t^p. Like traditional banks, shadow banks purchase Treasury futures with underlying value $\overline{f}_t = -\overline{b}_t$ to fully hedge their duration risk exposure and incur a transaction cost when purchasing or selling Treasury bonds.

Households Households maximize their lifetime expected utility of consumption and liquidity benefits:

$$\max_{\{c_u^h, e_u^h \geq 0, d_u^h \geq 0, p_u^h \geq 0, f_t^h\}_{u=t}^{\infty}} \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} dc_u^h \right] + \mathbb{E}_t \left[\int_t^{\infty} e^{-\rho(u-t)} \beta \log(h(d_u^h, p_u^h, \alpha_u)) du \right], \tag{16}$$

the law of motion of net worth:

$$dn_t^h = (e_t^h r_t^e + d_t^h r_t^d + p_t^h r_t^{pt} + f_t^h (r_t^b - r_t^\delta)) dt + f_t^h \sigma_t^b dZ_t - d\tau_t + d\pi_t + d\overline{\pi}_t + \nu |db_t| - dc_t^h, \tag{17}$$

[13]The allocation of transaction costs across parties has no bearing on our results.

[14]While our primary interpretation of shadow banks is that of relative-value hedge funds, the concept extends naturally to other institutions engaged in leveraged Treasury arbitrage via repo markets, such as liability-driven entities providing duration leverage to pension funds. This broader interpretation also encompasses securities dealers themselves, allowing us to assume without loss of generality that dealers exclusively intermediate repos rather than holding Treasuries outright. Under this framework, a dealer directly holding Treasuries can equivalently be represented as a combination of a pure repo intermediary and a shadow bank.

and the balance sheet constraint:

$$e_t^h + d_t^h + p_t^h = n_t^h + \tau_t^h. \tag{18}$$

Households choose their portfolio holdings of equity e_t^h, deposits d_t^h, and repos p_t^h given their liquidity preference α_t. They pay lump-sum taxes, $d\tau_t$, receive profits from traditional and shadow banks, $d\pi_t$ and $d\overline{\pi}_t$, and are rebated the bid-ask spread from Treasury transactions $\nu|db_t|$. They also purchase Treasury futures with underlying value $f_t^h = b_t - \underline{b}_t$ to fully hedge their duration risk exposure from tax liabilities.[15]

1.3 Discussion of Assumptions

We discuss the four key features in our framework and how those relate to existing institutions: the leverage ratio, the intraday liquidity requirement, and households' demand for repos and deposits, and a transaction cost for trading Treasuries.[16]

Leverage Ratio In our model, banks are subject to the leverage ratio constraint (11), which restricts the amount of leverage they can undertake. This assumption reflects real-world regulations, such as the supplementary leverage ratio (SLR) requirement of 6% imposed on large and systemically important banks in the U.S. As demonstrated below, this constraint consistently binds for banks because households' liquidity preference ensures that funding portfolios through deposits is always more cost-effective than using equity. Consequently, banks face a comparative disadvantage when holding Treasuries relative to shadow banks, which are exempt from such regulation. In other words, banks apply a higher funding value adjustment (Andersen, Duffie, and Song, 2019) compared to shadow banks, reflecting their requirement to finance part of their liabilities with expensive equity.[17]

Intraday Liquidity Requirement Following inequality (10), the intraday liquidity constraint restricts traditional banks' ability to lend in repo to a multiple of the central bank reserves

[15]To facilitate exposition, we assume that households do not directly hold Treasuries but rather hedge the duration of their tax liabilities only through futures. This condition would be guaranteed in equilibrium for a parameter β that is high enough. Our results are qualitatively robust to relaxing this assumption, provided that households have a strong preference for money-like assets over Treasuries.

[16]Capital and liquidity regulations were introduced after the 2008 crisis through U.S. legislation, notably the Dodd–Frank Act, and international initiatives like the Basel III framework, to curb excessive risk-taking by systemic institutions. As our goal is a positive theory of regulation's impact on Treasury markets, we abstract from the underlying frictions motivating these regulations.

[17]For alternative micro-foundations leading to similar balance sheet costs, we refer to the modified-CAPM model by Frazzini and Pedersen (2014), where leverage constraints incentivize investors to avoid low-beta investments, such as low-yield, risk-free arbitrage opportunities, in favor of high-beta assets needed to achieve optimal risk exposure. Alternatively, Andersen, Duffie, and Song (2019) demonstrate that leverage ratio constraints generate a debt-overhang cost, resulting in increased hurdle rates from shareholders to fund arbitrage trades.

they hold. This constraint captures an important piece of the Basel III regulatory infrastructure (implemented in the U.S. under Regulation YY), requiring banks to pre-fund in reserves the entirety of their anticipated gross outflows for the day. Under such a requirement, regulators expect that banks do not make use of intraday reserves overdraft at the Fed; as doing so would indicate that banks did not hold enough reserves to cover their intraday settlements and failed to meet their obligation. A direct consequence of this piece of regulation is that the amount of repo lending that banks can provide in a given day is restricted by the reserves they hold to settle it. d'Avernas, Han, and Vandeweyer (2025) provide a full structural microfoundation of this constraint, which we distill to its essential mechanism here. A bank enters the day with reserves m_t and settles repo in real time; over the course of the day the reserves it lends out recirculate—borrowers' payments are redeposited and settle further transactions—so that, across the day's settlement rounds, the bank can sustain a repo book that is a multiple of its reserve stock. Subject to Regulation YY's no-overdraft requirement in each round, this delivers constraint (10), with $\kappa = N(1 - \bar{\varepsilon})$ the product of the intraday reserve velocity N (the number of settlement rounds in which reserves recirculate) and the per-round buffer $1 - \bar{\varepsilon}$ against the worst-case payment outflow—a pure payment-system and regulatory parameter, independent of the bank's balance sheet and of the household preference state, and exceeding one when reserves turn over more than once within the day. We refer to Correa, Du, and Liao (2020); Copeland, Duffie, and Yang (2025); and Pozsar (2019) for further discussions of this intraday liquidity constraint and its implications for banks' ability to lend in repo markets.

Imperfect Substitution Between Repo and Deposit In reality, households and firms hold cash-like assets largely in the form of bank deposits and money market fund shares. Since the reform of money markets in 2016, most money market fund shares have been invested in either triparty repos with dealers or in short-duration Treasury securities and close substitutes (Cipriani and La Spada, 2021). In our model, we abstract from money market funds and assume a perfect pass-through from money market fund share demand to repo.[18] The imperfect substitutability between repo and deposits captures the fact that deposits and money market fund shares have important diverging characteristics. For instance, deposits are more liquid, have lower interest, and are riskier when held above FDIC insurance quantity limits and safer below this limit. Empirically, Krishnamurthy and Li (2023) estimate this elasticity of substitution to be smaller than one on average, consistent with our imperfect substitute assumption.

Treasury Transaction Costs Although transaction costs associated with trading Treasury securities are lower than for most other capital market instruments, they remain substantial for frequent trading. Bid-ask spreads for on-the-run securities typically lie in the range of 0.5 to 2.5 bps (Fleming, 2000). Evidence suggests even larger spreads for comparable off-the-run

[18] D'Avernas and Vandeweyer (2024) investigate the role of the supply of Treasury bills in driving triparty repo rates when banks face balance sheet costs, which is abstracted from in this work by assuming that households do not hold Treasuries directly.

Treasuries, with bid-ask spreads ranging from approximately 1 to 4 basis points (bps) under normal market conditions, making 2.5 bps a representative midpoint for a round-trip transaction (Investment Company Institute, 2022).[19]

1.4 Solving

Equilibrium Definition With the specified aggregate shocks, the economy's state space is fully characterized by the time-varying parameter α_t.[20] We define a Markov equilibrium in α_t as follows.

Definition 1. *A* ***Markov equilibrium*** $\mathcal{M}$ *in* α_t *is a set of functions* $g_t = g(\alpha_t)$ *for (i) interest rates* $\{r_t^b, r_t^\delta, r_t^m, r_t^p, r_t^{pt}, r_t^d, r_t^e\}$ *and (ii) individual controls for traditional banks* $\{b_t, m_t, f_t, p_t, x_t, d_t, e_t\}$*, shadow banks* $\{\overline{b}_t, \overline{f}_t, \overline{p}_t\}$*, and households* $\{e_t^h, d_t^h, p_t^h\}$ *such that:*

1. *Agents' optimal controls (ii) solve their respective problems given prices (i).*
2. *The balance sheet constraint of the central bank is satisfied.*
3. *The balance sheet constraint and budget constraint of the Treasury are satisfied.*
4. *Markets clear:*

(a) Treasury bonds:	$\overline{b}_t + b_t = b_t^\tau - \underline{b}_t,$
(b) futures:	$f_t + \overline{f}_t + f_t^h = 0,$
(c) reserves:	$m_t = \underline{m}_t,$
(d) triparty repo:	$p_t^h = x_t,$
(e) bilateral repo:	$x_t + p_t = \overline{p}_t,$
(f) deposits:	$d_t = d_t^h.$
(g) equities:	$e_t = e_t^h.$

5. *The law of motion for* α_t *is consistent with agents' expectations.*

[19] Despite appearing minor, a 2.5 bps round-trip cost corresponds to an annualized rate of $(1 - 0.00025)^{360} - 1 = -8.6\%$ when incurred daily. Thus, the effective cost of frequent rebalancing becomes comparable to repo funding differentials observed during episodes of market stress, significantly affecting arbitrage and execution strategies. In our quantitative exercise (Section 3), where we let the transaction cost vary with the outstanding Treasury supply, its level evaluated at the sample-mean supply, ν_0, is estimated at 1.3 bps.

[20] Aggregate Treasury portfolio weights are not aggregate state variables because the *marginal* cost of transactions is constant and not a function of the size of the transaction. Thus, if it is profitable to fire-sell Treasuries when entering state $\{\alpha_t, b_{t\text{-}}, \overline{b}_{t\text{-}}\}$, it is also profitable to fire-sell Treasuries when entering state $\{\alpha_t, b'_{t\text{-}}, \overline{b}'_{t\text{-}}\}$, for $b'_{t\text{-}} \neq b_{t\text{-}}$ and $\overline{b}'_{t\text{-}} \neq \overline{b}_{t\text{-}}$. Therefore, the Markov functions are such that $g(\alpha_t, b'_{t\text{-}}, \overline{b}'_{t\text{-}}) = g(\alpha_t)$ and the equilibrium is entirely determined by α_t. See Appendix Section A.

Equilibrium Restrictions Since our objective is to study disruptions in the Treasury market, we simplify our exposition by restricting attention to a subset of economically relevant equilibria.[21] Implicitly, this restriction constrains the set of parameters and policy scenarios considered. Specifically, we focus on equilibria characterized by: (i) a strictly positive supply of reserves ($\underline{m}_t > 0$); (ii) a steady-state configuration α^s in which traditional banks hold Treasuries ($b(\alpha^s) > 0$) and are net borrowers in repos ($p(\alpha^s) < 0$) in steady state; (iii) household balance sheets exceed the quantity of repo financing supplied by traditional banks ($\tau_t^h > \kappa m_t$); (iv) and there exists a threshold $\alpha' \in (\underline{\alpha}, 1)$ above which shadow banks optimally choose to sell Treasuries.

Value Functions In Appendix A, we guess and verify that the value functions for traditional and shadow banks take the following form:

$$V(b;\alpha) = \xi(\alpha) + \theta(\alpha)b, \qquad \overline{V}(\bar{b};\alpha) = \bar{\xi}(\alpha) + \bar{\theta}(\alpha)\bar{b}. \tag{19}$$

To solve for the marginal value of holding Treasuries, $V_b(b;\alpha) = \theta(\alpha)$ and $\overline{V}_b(\bar{b};\alpha) = \bar{\theta}(\alpha)$, we apply the envelope theorem to the Hamilton-Jacobi-Bellman equations:

$$(\rho + \lambda')\bar{\theta}(\alpha') = r^\delta(\alpha') - r^p(\alpha') + \lambda'\nu\,\text{sign}(\bar{b}(\alpha^s) - \bar{b}(\alpha')), \tag{20}$$
$$(\rho + \lambda')\theta(\alpha') = r^\delta(\alpha') - r^d(\alpha') + \lambda'\nu\,\text{sign}(b(\alpha^s) - b(\alpha')) - \phi\vartheta^e(\alpha'), \tag{21}$$

for α' such that $\bar{b}(\alpha') \neq \bar{b}(\alpha^s)$ and where $\vartheta^e(\alpha)$ is the shadow price of the LR constraint (11).[22] This first condition implies that, following a shock large enough to trigger a fire sale, the marginal value of reducing its Treasury holdings for shadow banks equals the Treasury-repo spread—the immediate gain from unwinding—plus the expected transaction costs associated with returning to the steady state in the future. Analogously for traditional banks, in the shock state, the marginal benefit of purchasing Treasuries equals the Treasury-deposit spread—reflecting the marginal cost of financing—plus the expected transaction costs of returning to the steady state, adjusted for the shadow value of the leverage-ratio constraint when it binds.

[21] These restrictions delimit the empirically relevant region of the parameter space on which our results are stated—in particular, restriction (iv) below confines the analysis to the scarce-reserves region in which fire sales can arise, the case of interest for studying disruptions; Proposition 6 characterizes the complementary abundant-reserves region. Restrictions (iii) and (iv) and the Treasury-holding part of restriction (ii) are formalized as Assumptions 1, 2 and 4 in the Appendix, together with a regularity condition on the dealer subsidiary (Assumption 3); they reduce the number of distinct cases we need to examine within the proofs.

[22] See Appendix A for the remaining cases.

First-order Conditions The first-order conditions for reserves, bilateral repo, triparty repo, and equity of traditional banks are given by

$$r^m(\alpha) - r^d(\alpha) = \phi\vartheta^e(\alpha) - \kappa\vartheta^m(\alpha), \tag{22}$$

$$r^p(\alpha) - r^d(\alpha) = \begin{cases} \phi\vartheta^e(\alpha) + \vartheta^m(\alpha) & \text{if } p(\alpha) > 0, \\ \in [0, \phi\vartheta^e(\alpha) + \vartheta^m(\alpha)] & \text{if } p(\alpha) = 0, \\ 0 & \text{if } p(\alpha) < 0, \end{cases} \tag{23}$$

$$r^p(\alpha) - r^{pt}(\alpha) = \phi\vartheta^e(\alpha), \tag{24}$$

$$r^e(\alpha) - r^d(\alpha) = \vartheta^e(\alpha), \tag{25}$$

where $\vartheta^m(\alpha)$ is the shadow price of the IL constraint (10). In equation (22), traditional banks choose their reserve holdings by equating the spread between reserve and deposit rates to the balance-sheet costs associated with tightening the LR constraint with a larger balance sheet, $\phi\vartheta^e(\alpha)$, and relaxing the IL constraint by adding more liquidity, $\kappa\vartheta^m(\alpha)$. Similarly, in equation (23), when a traditional bank lends in repos ($p_t > 0$), it will do so up to the point at which the bilateral repo rate exactly compensates for the tightening of the IL constraint. When a traditional bank funds itself in repo ($p_t < 0$), repos serve as a substitute for deposits, and the repo rate must therefore equal the deposit rate because any deviation from this condition would constitute an arbitrage. For repo spreads falling within the range of those two values, traditional banks neither borrow nor lend in the repo market. In equation (24), traditional banks require a spread between bilateral and triparty repo to compensate for the balance sheet cost incurred by intermediating repo at the dealer subsidiary. Finally, substituting one unit of deposits with equity relaxes the LR constraint by exactly one unit of slack; hence, in (24), the equity–deposit spread equals the shadow price of the LR constraint, $\vartheta^e(\alpha)$.

In addition, households' first-order condition for their relative holdings of triparty repo and deposit satisfies

$$r^{pt}(\alpha) - r^d(\alpha) = \beta\left(\frac{\alpha}{d^h(\alpha)} - \frac{1-\alpha}{p^h(\alpha)}\right). \tag{26}$$

Households equalize the marginal benefit of investing in triparty repo over deposits, as given by the spread between the rates on these two assets, to the marginal convenience cost of reallocating one unit of wealth from deposits to repo.

The first-order condition for households' investment in bank equity is given by

$$r^e(\alpha) - r^d(\alpha) = \beta\frac{\alpha}{d^h(\alpha)}. \tag{27}$$

Equation (27) states that the equity-deposit spread equals the marginal convenience yield of deposits, $\beta\alpha/d^h(\alpha)$, which captures the liquidity value households assign to deposits in the Cobb–Douglas aggregator and, in equilibrium, determines the shadow price of the LR constraint.

Together with the traditional bank first-order condition for equity, this equation immediately implies that the LR constraint is always binding.

Furthermore, the envelope condition for households' net worth pins down the return on equity as

$$r^e(\alpha) = \rho. \tag{28}$$

Lastly, the first-order condition for Treasury portfolio weights yields

$$\nu \,\mathrm{sign}(b(\alpha) - b(\alpha_-)) = \theta(\alpha) \qquad \text{if } b(\alpha) \neq b(\alpha_-), \tag{29}$$

$$\nu \,\mathrm{sign}(\overline{b}(\alpha) - \overline{b}(\alpha_-)) = \overline{\theta}(\alpha) \qquad \text{if } \overline{b}(\alpha) \neq \overline{b}(\alpha_-). \tag{30}$$

According to this equation, both traditional and shadow banks trade off the marginal cost of a transaction, on the left-hand side, against the marginal benefit of purchasing or selling Treasury bonds, on the right-hand side. The sign function reflects the fact that to sell bonds, the marginal value of holding an additional unit of bond needs to be negative (and larger than the marginal transaction cost), and conversely for purchases. The value functions depend on both current and future rates, as well as the stochastic process governing the state variable. For example, as we show in the following section, the marginal value of holding Treasuries for shadow banks increases when the cash-futures basis $r^\delta(\alpha) - r^p(\alpha)$ is larger or when the arrival intensity of a disruption λ is lower.

2 Theoretical Results

This section presents the paper's main theoretical results. A key challenge in characterizing the equilibrium arises because repo and Treasury market conditions during the shock state are determined by the endogenous portfolio positions set in the steady state, which themselves depend on agents' expectations about future market conditions. Consequently, an equilibrium corresponds to a fixed point where prices and portfolio allocations are jointly consistent across states. We first derive two preliminary results necessary for equilibrium characterization within our model. We then study how the model adjusts to shocks in the crisis state. Finally, we derive a series of results to study how varying model parameters affect equilibrium fragility in repo and Treasury markets. Proofs of all propositions are provided in Appendix B. We use $z(\alpha^s)$ and $z(\alpha')$ to denote a variable z in the steady state and the shock states, respectively, and $z(\alpha)$ or z_t to denote when the statement holds at all times.

2.1 Preliminaries

In this section, we derive two results characterizing the equilibrium in our model as important preliminaries for the rest of the analysis.

Uniqueness and Inaction We demonstrate in Proposition 1 that the equilibrium is unique in all quantities and prices except for the cash-futures implied rate r_t^δ. This rate r_t^δ can take a continuum of values within boundaries because of the presence of transaction costs when trading treasuries, which creates an inaction region within which no Treasuries are traded. Within these bounds, a deviation in the cash-futures basis is not sufficient to justify paying the transaction cost, so agents do not rebalance their portfolios.

Proposition 1. *The equilibrium is unique in prices* $\{r^m, r^p, r^{pt}, r^d, r^e\}$ *and quantities* $\{b, f, m, p, x, d, e, \overline{b}, \overline{f}, \overline{p}, e^h, d^h, p^h\}$ *as functions of* $\alpha \in [\underline{\alpha}, 1)$. *Also, the cash-futures implied risk-free rate is bounded by* $\underline{r}^\delta(\alpha) \leq r^\delta(\alpha) \leq \overline{r}^\delta(\alpha)$, *where*

$$\underline{r}^\delta(\alpha') \equiv \max\{r^d(\alpha') + \phi\vartheta^e(\alpha') - \rho\nu, r^p(\alpha') - (\rho + 2\lambda')\nu\}$$
$$\overline{r}^\delta(\alpha') \equiv \min\{r^d(\alpha') + \phi\vartheta^e(\alpha') + (\rho + 2\lambda')\nu, r^p(\alpha') + \rho\nu\}$$

for $\alpha' \in [\underline{\alpha}, 1)$.[23]

The boundaries of the inaction region in Proposition 1 capture the opportunity costs associated with buying or selling Treasuries for either traditional or shadow banks. The max and min operators ensure that, in equilibrium, neither institution is willing to sell (lower bound) or purchase (upper bound) Treasuries outside of the fire-sale region. When α' is sufficiently large to trigger sales by shadow banks, $\underline{r}^\delta(\alpha') = r^\delta(\alpha') = \overline{r}^\delta(\alpha')$. In what follows, $r^\delta(\alpha')$ denotes the midpoint between the upper and lower bounds specified in Proposition 1.

2.2 Shock State Dynamics

Before analyzing results that depend on the model's fixed-point between steady and shock states, we first characterize the dynamics within the shock state.

Inelastic Benchmark To build intuition, we first analyze how the model accommodates a repo shock in a simplified version where we constrain the portfolios of both traditional and shadow banks to be fully inelastic. In such a case, the financial sector does not provide any elasticity in quantities, and all adjustments occur entirely through changes in prices. Combining

[23] See the proof in Appendix B for equivalent boundaries at $\alpha = \alpha^s$.

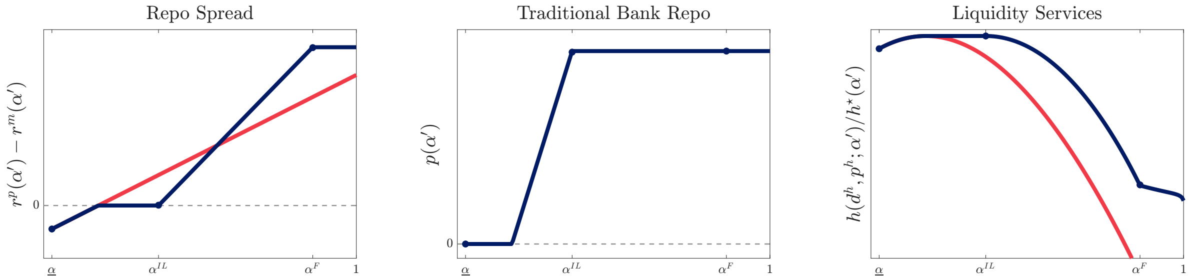


Figure 4: Inelastic Benchmark and Crisis State Dynamics. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $p(\alpha')$, and the ratio of liquidity services to its optimum $h(d^h, p^h; \alpha')/h^\star(\alpha')$ where $h^\star(\alpha') \equiv \max_p h(\tau^h(1-\phi) - p, p; \alpha')$. We plot these variables under two different settings. In red, we provide an inelastic benchmark with no repo lending or borrowing by traditional banks, nor Treasury sales. In blue, we provide the solution of the complete framework, in which banks can adjust their balance sheets.

equations (22), (24), and (26), the repo-to-reserves spread can be written in the inelastic case as

$$r^p(\alpha') - r^m(\alpha') = \beta \left(\frac{\alpha'}{d^h} - \frac{1-\alpha'}{p^h} \right), \tag{31}$$

where the allocations d^h and p^h are held constant. That is, the repo-to-reserves spread reflects households' relative preference between repos and deposits: when households' preference shifts away from repos toward deposits—as captured by a positive shock to the parameter α'—repo rates must rise to incentivize households to continue holding repos. This scenario is illustrated by the red lines in the three panels of Figure 4. Since banks do not adjust their repo lending (middle panel), repo spreads must increase with the magnitude of the shock α' (left panel) to compensate households for lower liquidity benefits (right panel).

We now turn to examining how the model responds to liquidity shocks when banks' balance sheets have some flexibility, yet traditional banks are subject to both balance-sheet costs and the intraday liquidity constraint.

Shock Dynamics under Constraints When banks' portfolio allocations are flexible, the dynamics of price and quantity adjustments become more intricate, as they depend on whether the shock is large enough for the IL constraint to bind or for Treasury sales to be optimal. The blue lines in Figure 4 depict these dynamics upon entering the shock state.

As long as the IL constraint remains slack ($\alpha' < \alpha^{IL}$), the repo-to-reserves spread $r^p(\alpha') - r^m(\alpha')$ is bounded above at zero (left panel), since banks would otherwise arbitrage any positive spread by substituting reserves for repo, as implied by first-order conditions 22 and 23. Banks' capacity to lend in repo markets (middle panel) allows households to maintain their preferred mix of repo and deposits (right panel) until the IL constraint becomes binding.

At the threshold α^{IL}, banks can no longer expand repo lending. Households must then absorb a suboptimal mix of repos and deposits (right panel), which in turn leads to an increase in repo spreads (left panel). Once the IL constraint binds, the repo-to-reserves spread increases at a faster rate compared to the inelastic benchmark, reflecting a liquidity premium for reserves driven by the IL constraint's shadow cost $\vartheta^m(\alpha')$ in equation (22).

Finally, when the shock magnitude exceeds α^F, shadow banks find it optimal to incur the transaction cost and liquidate Treasuries to traditional banks rather than continue funding these positions in repo at a stressed rate. These Treasury sales provide shadow banks with an outside option, which caps the extent to which the repo rate can spike (left panel).

A Condition for Repo Spikes The next lemma characterizes the condition under which the repo rate spikes above the interest on reserves.

Lemma 1. *$r^p(\alpha') > r^m(\alpha')$ if and only if $\alpha' > \alpha^{IL}$, where α^{IL} is such that*

$$\underbrace{b^\tau - \underline{b} - b(\alpha^{IL})}_{\textit{repo demand}} = \underbrace{(1-\alpha^{IL})\tau^h(1-\phi) + \kappa\underline{m}}_{\textit{maximum repo capacity}}. \tag{32}$$

Lemma 1 establishes a necessary and sufficient condition under which the bilateral repo rate (r_t^p) exceeds the interest rate on reserves (r_t^m). The left-hand side of Equation (32) represents the quantity that shadow banks must finance. On the right-hand side, households can supply at most $(1-\alpha_t)\tau^h$ at the Cobb-Douglas liquidity aggregator's maximum, which translates into an effective repo supply of $(1-\alpha_t)\tau^h(1-\phi)$ once the equity requirement ϕ in the LR constraint is taken into account. Traditional banks can also add up to $\kappa\underline{m}$ repo lending before the IL constraint becomes binding. When the inequality is satisfied, the IL constraint binds, and market clearing requires $r_t^p > r_t^m$ to attract marginal household funding into the repo market. This condition characterizes the threshold α^{IL}. Conversely, if the inequality does not hold, the combined supply is sufficient to prevent a spike in the repo rate ($r_t^p \leq r_t^m$).

A Condition for Treasury Sales Similarly, the following lemma defines the repo-rate threshold for α' that triggers Treasury fire sales.

Lemma 2. *$\bar{b}(\alpha') < \bar{b}(\alpha^s)$ if and only if $\alpha' > \alpha^F$, where α^F is such that*

$$\left(1 + \frac{\lambda'}{\rho + \lambda'}\right)\nu = \frac{r^p(\alpha^F) - r^\delta(\alpha^F)}{\rho + \lambda'}. \tag{33}$$

Lemma 2 establishes a necessary and sufficient condition under which shadow banks sell Treasuries upon entering in the shock state α'. The threshold Equation (33) corresponds to the shock size at which shadow banks' cost of liquidating Treasuries—incurring a transaction cost ν— is equal to the discounted expected loss from holding Treasuries when the repo rate rises above the interest on reserves during the shock state. The left-hand side captures the present value of paying the transaction cost both immediately and upon returning to the steady state,

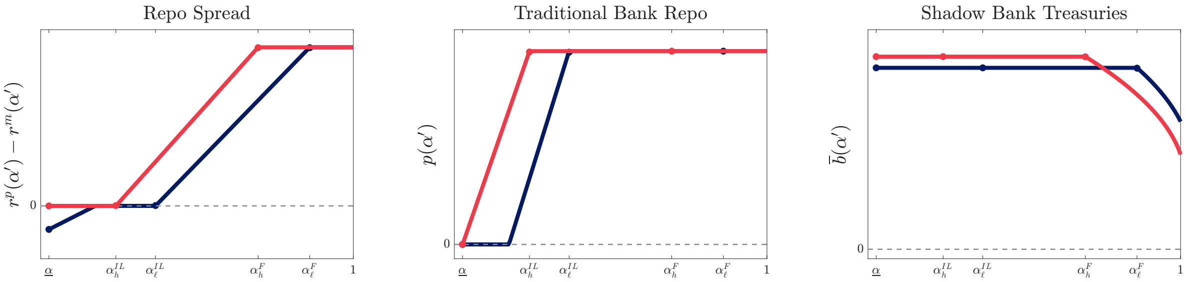


Figure 5: Regulation and Risk-Taking. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $p(\alpha')$, and shadow banks' Treasury holdings $\bar{b}(\alpha')$ for low (blue) and high (red) regulatory ratio requirement ϕ. Subscripts ℓ and h rank each pair of thresholds by size ($\alpha_\ell^{IL} < \alpha_h^{IL}$ and $\alpha_\ell^F < \alpha_h^F$); the markers on each curve indicate the thresholds of the corresponding economy.

while the right-hand side reflects the expected discounted carry loss over the duration of the shock.

2.3 Equilibrium Results

In this section, we present the paper's main results, illustrating how changes in key parameters of our dynamic model influence equilibrium prices and quantities, starting with the effect of banks' leverage ratio regulation. Throughout, we maintain the equilibrium restrictions introduced above; in particular, the comparative statics below apply to the scarce-reserves region, $\underline{m} \leq \tau^h/(1+\kappa)$, in which repo spikes and fire sales can arise in equilibrium (Assumption 1). Proposition 6 characterizes the complementary abundant-reserves region, in which the intraday liquidity constraint never binds.

Regulatory Arbitrage and Treasury Holdings We first characterize how the steady state holdings of Treasuries and endogenous risk are affected by changes in the balance sheet cost through Proposition 2.

Proposition 2. *If $\sqrt{\lambda/\big((1-\underline{\alpha})(\rho+\lambda')\big)} < \kappa\underline{b}/b^\tau$, a tighter leverage ratio constraint ϕ results in larger shadow banks' Treasury holdings in the steady state, a higher probability of fire sales, and higher repo rate spikes. That is,*

$$\frac{\partial \bar{b}(\alpha^s)}{\partial \phi} > 0, \quad \frac{\partial \mathbb{P}\big(\bar{b}(\alpha') < \bar{b}(\alpha^s)\big)}{\partial \phi} > 0, \quad \textit{and} \quad \frac{\partial (r^p(\alpha') - r^m(\alpha'))}{\partial \phi} > 0.$$

Proposition 2 highlights a potential unintended consequence of leverage regulations: tightening banks' regulatory leverage ratios can shift Treasury holdings to the unregulated shadow banking sector, thereby amplifying Treasury market fragility and liquidity risk.[24] Shadow banks have

[24]This mechanism is consistent with the empirical evidence in Allahrakha, Cetina, and Munyan (2018) and Barth and Kahn (2025).

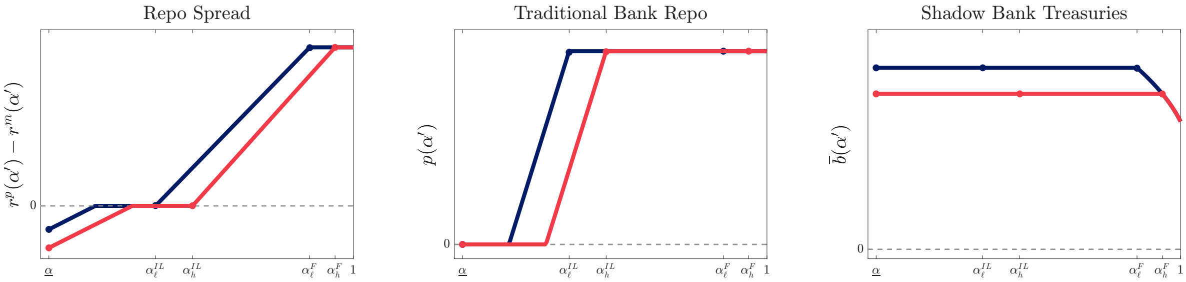


Figure 6: Shock Frequency and Intensity. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $p(\alpha')$, and shadow banks' Treasury holdings $\bar{b}(\alpha')$ for low (blue) and high (red) Poisson arrival intensity λ (probability of a flight-to-deposit shock). Subscripts ℓ and h rank each pair of thresholds by size ($\alpha_\ell^{IL} < \alpha_h^{IL}$ and $\alpha_\ell^F < \alpha_h^F$); the markers on each curve indicate the thresholds of the corresponding economy.

a comparative advantage in issuing repos to households backed by duration-hedged Treasuries, as they can create one dollar of repo for each dollar of Treasuries held. In contrast, traditional banks must finance Treasuries with a fraction ϕ of equity, rendering them less efficient in repo intermediation.

An increase in the leverage ratio requirement ϕ reinforces this comparative advantage. This mechanism operates when traditional banks are net repo borrowers in the steady state ($p(\alpha_s) < 0$). In that case, the leverage-ratio constraint pins down the benefit of repo borrowing, $r^e(\alpha_s) - r^p(\alpha_s) = \phi\vartheta^e(\alpha_s)$, so an increase in ϕ lowers the bilateral repo rate $r^p(\alpha_s)$ and pushes this funding advantage further negative. This cheapening of repo funding for shadow banks makes the cash-futures basis trade more attractive for those institutions, thereby incentivizing them to purchase more Treasuries, as illustrated in Figure 5.

A second force is, however, also at play: a tighter leverage constraint also makes the intraday liquidity constraint bind more easily in the shock state, thereby increasing shadow banks' funding risks. Formally, in Lemma 1, a higher leverage ratio ϕ reduces the household term $(1-\alpha_t)\tau^h(1-\phi)$ on the right-hand side, thereby lowering the repo-supply capacity and making the inequality easier to satisfy. Thus, the economy hits the constrained region (where $p = \kappa\underline{m}$) for smaller shocks α'. This heightened liquidity risk incentivizes shadow banks to reduce their positions in the cash-futures basis and hold fewer Treasuries. The parameter condition in Proposition 2 ensures that this increase in the likelihood of stress remains moderate enough for the first, regulatory-arbitrage force to dominate, so that shadow banks' Treasury holdings increase with the intensity of banks' leverage ratio.

Volatility Paradox in Treasury Markets In Proposition 3, we formally characterize the impact of variations in the probability of a flight-to-deposit shock on equilibrium portfolio allocations and the endogenous dynamics of Treasury market fragility.

Proposition 3. *A lower shock arrival intensity λ results in larger shadow banks' Treasury* *holdings in the steady state, a higher probability of fire sales, and higher repo rate spreads to interest on reserves. That is,*

$$\frac{\partial \bar{b}(\alpha^s)}{\partial \lambda} < 0, \quad \frac{\partial \mathbb{P}\big(\bar{b}(\alpha') < \bar{b}(\alpha^s)\big)}{\partial \lambda} < 0, \quad \textit{and} \quad \frac{\partial (r_t^p - r_t^m)}{\partial \lambda} < 0.$$

Proposition 3 highlights the endogenous response of shadow banks' cash-futures positions to changes in the likelihood of entering into the shock state. This response reflects a trade-off between the carry on the cash-futures position and the losses that materialize upon the arrival of a flight-to-deposits shock. In steady state, the shadow banks' marginal value of holding additional Treasuries equals the current carry, $r^\delta - r^p$, minus an expected penalty proportional to the shock arrival rate λ, which captures anticipated adjustment costs and funding losses during the shock state. A decrease in λ reduces this expected penalty, thereby raising the net marginal value of holding Treasuries and incentivizing shadow banks to expand their repo-financed Treasury positions.

Consequently, when shocks occur, larger steady-state Treasury holdings translate into greater repo financing needs, pushing repo rates higher to clear the market. These higher repo rates are more likely to trigger Treasury sales, amplifying both the probability and severity of market disruptions. This heightened sensitivity to shocks manifests as a leftward shift in both the threshold α^{IL} that triggers a repo spike and the threshold α^F that triggers Treasury sales in Figure 6.

This proposition echoes the "volatility paradox" of Brunnermeier and Sannikov (2014a): a lower frequency of shocks encourages greater ex-ante risk-taking, thereby increasing the severity of realized disruptions. This logic further highlights a policy trade-off: discretionary support during stress episodes results in rare but severe disruptions, whereas a standing repo facility—by systematically increasing effective repo supply (the right-hand side in Lemma 1)—can limit the build-up of risk and reduce the magnitude of disruptions. This result also highlights the importance of using a dynamic model when calculating counterfactual scenarios, due to the strong adaptation of shadow banks to prevailing risk conditions, which is the purpose of Section 3.

Crisis Duration and Dynamics We now investigate the role of investor expectations about the duration of the shock in determining equilibrium prices and portfolio allocations within the repo and Treasury markets in Proposition 4.

Proposition 4. *A longer shock duration (a lower λ') results in smaller shadow banks' Treasury holdings in the steady state, lower repo spreads to interest on reserves, and a higher cash-futures basis. That is,*

$$\frac{\partial \bar{b}(\alpha^s)}{\partial \lambda'} > 0, \quad \frac{\partial (r_t^p - r_t^m)}{\partial \lambda'} > 0, \quad \textit{and} \quad \frac{\partial (r_t^\delta - r_t^p)}{\partial \lambda'} < 0.$$

Proposition 4 shows that longer-lasting shocks (i.e., a lower λ') lead to smaller repo rate spikes

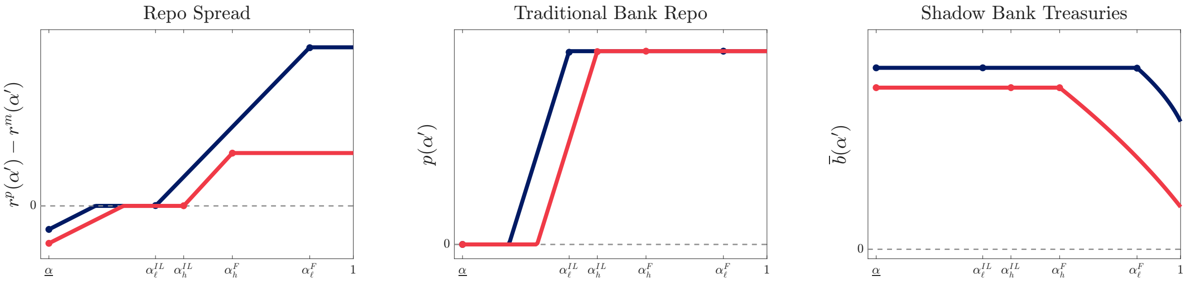


Figure 7: Shock Duration and Shock State Dynamics. This figure shows equilibrium repo spreads $r^p(\alpha') - r^m(\alpha')$, traditional banks' repo lending $p(\alpha')$, and shadow banks' bond holdings $\bar{b}(\alpha')$ for low (red) and high (blue) Poisson arrival intensity λ' (probability of exiting the shock state and returning to steady state). Subscripts ℓ and h rank each pair of thresholds by size ($\alpha_\ell^{IL} < \alpha_h^{IL}$ and $\alpha_\ell^F < \alpha_h^F$); the markers on each curve indicate the thresholds of the corresponding economy.

but a larger widening of the cash-futures basis and Treasury sales. This result is illustrated in Figure 7: a higher λ'—a shorter expected duration of the shock—raises the negative spread that shadow banks are willing to endure before liquidating Treasuries. When a repo supply shock is anticipated to be temporary (high λ'), shadow banks accept temporarily higher repo rates for a short period to avoid the transaction costs associated with liquidation, thereby reducing the likelihood of fire sales. Conversely, when the shock is expected to persist (low λ'), α^F is pushed to the left, and shadow banks prefer to liquidate their Treasury holdings rather than incur prolonged elevated repo financing costs. As a consequence, increased shock persistence raises the steady-state cash-futures basis and induces shadow banks to reduce their steady-state Treasury holdings.

Thus, Proposition 4 offers a potential explanation for why repo rate spikes tend to be particularly pronounced at quarter-end, when market participants anticipate the shock to be short-lived and mean-revert quickly, typically within a day. It also sheds light on the observed asymmetry between the dramatic repo spike in September 2019—when Treasury markets remained relatively stable—and the contrasting episode of March 2020, when Treasury yields surged sharply with only a moderate increase in repo rates. In our framework, this difference arises because hedge funds likely perceived the September shock as transitory, whereas the March disruption was expected to persist longer. The next section uses the calibrated model to provide quantitative insights consistent with this interpretation.

Central Bank Balance Sheet and Disruption Likelihood We next consider how the overall size of the central bank balance sheet affects Treasury market fragility.

Proposition 5. *In the scarce-reserves regime ($\underline{m} \leq \tau^h/(1+\kappa)$, implied by Assumption 1), a smaller central bank balance sheet $\underline{b}$ results in smaller shadow banks' Treasury holdings in the* *steady state, a larger probability of fire sales, and a higher Treasury cash-futures basis. That is,*

$$\frac{\partial \bar{b}(\alpha^s)}{\partial \underline{b}} > 0, \quad \frac{\partial \mathbb{P}\big(\bar{b}(\alpha') < \bar{b}(\alpha^s)\big)}{\partial \underline{b}} < 0, \quad \textit{and} \quad \frac{\partial (r^\delta(\alpha^s) - r^p(\alpha^s))}{\partial \underline{b}} < 0.$$

This result follows from Lemma 1: a contraction of the central bank's balance sheet (QT) simultaneously tightens both sides of the repo market. On the asset side, a reduction in the central bank's Treasury holdings leaves a larger share of securities to be financed by traditional and shadow banks, thereby increasing funding demand. On the liability side, fewer reserves diminish banks' intraday liquidity capacity, lowering the ceiling on repo supply. Consequently, both the likelihood and the magnitude of repo rates exceeding the interest on reserves in the shock state rise unambiguously as the central bank's balance sheet contracts. These larger and more frequent spikes expand the set of shocks in which fire sales occur. Anticipating the resulting increase in expected funding losses, hedge funds demand greater ex-ante compensation, which manifests as a higher steady-state cash-futures basis, and hold less Treasuries in equilibrium.

Central Bank Balance Sheet and Liquidity Provision Finally, Proposition 6 characterizes how the supply of reserves influences the efficiency with which the financial system provides liquidity services to households.

Proposition 6. *If $\underline{m} > \tau^h/(1+\kappa)$, no fire sales occur and reserves provide no further liquidity benefits; moreover, for $\underline{m}$ sufficiently large, additional units of central bank reserves strictly decrease net-of-transaction-costs liquidity benefits.*

Proposition 6 highlights the presence of a trade-off in the decision about the central bank balance sheet size. When reserves are abundant—specifically, once they exceed a fixed fraction $1/(1+\kappa)$ of the outstanding Treasury supply, $\underline{m} > b^\tau/(1+\kappa)$ (with $b^\tau = \tau^h$ in equilibrium)—the intraday-liquidity (IL) constraint never binds, and the repo market consistently clears at $r^p \leq r^m$, thereby eliminating repo rate spikes. In this abundant-reserve region, shadow banks no longer have incentives to liquidate Treasuries, hence avoiding fire sales and associated transaction costs altogether. Thus, any additional reserves beyond this threshold provide no further marginal liquidity benefits, as they neither relax binding constraints nor mitigate losses during shock states. Instead, additional reserves merely expand traditional banks' balance sheets, amplifying regulatory balance-sheet costs by tightening LR constraints and crowding out efficient intermediation capacity.[25] Conversely, when reserves are scarce, the IL constraint binds frequently, causing frequent repo spikes and fire-sale losses. In this scenario, expanding the central bank's balance sheet yields first-order liquidity benefits. Together, these considerations imply a non-monotonic relationship between reserves and liquidity provision: liquidity benefits rise initially as reserves increase and the IL constraint relaxes, but eventually decline beyond an

[25] This mechanism is analogous to Diamond et al. (2024), who demonstrate that quantitative easing can crowd out bank lending by increasing regulatory costs.

optimal threshold, identifying an interior reserve level that balances these opposing effects.[26]

Proposition 6 thus yields a concrete prescription for central-bank reserve-supply policy. Targeting reserves just above the threshold, $\underline{m}_t > b_t^\tau/(1+\kappa)$, ensures that the IL constraint remains slack for every realization of the household preference state α, eliminating repo rate spikes for any shock; the rule indexes the central bank's balance sheet to the Treasury supply, with the ratio $1/(1+\kappa)$ determined by the regulatory intraday-liquidity parameter. The rule is conservative—it provisions reserves for the worst-case stress state and therefore incurs the leverage-ratio costs noted above outside stress episodes. A state-contingent refinement $\underline{m}_t(\alpha_t)$, tailoring reserves to the current state so that the IL constraint just remains slack at each α, would deliver the same spike prevention with a smaller average central-bank balance sheet. Operationally, the Federal Reserve's Standing Repo Facility and emergency open-market operations implement a version of this state-contingent injection, supplementing a leaner ex-ante reserve stance. We next turn to the quantitative analysis.

3 Quantitative Exercise

In this section, we calibrate and estimate the model to assess its empirical fit, analyze its implications for repo and Treasury markets, and conduct counterfactual exercises.

3.1 Calibration and Estimation Procedure

We calibrate the Poisson arrival rate λ to 0.069 events per trading day in order to match the observed frequency of repo-rate spikes in our sample: 139 days on which the spread between the general-collateral repo rate and the interest on reserve balances (IORB) exceeds 10 basis points over January 2017 – February 2025, corresponding to approximately 17 events per year and concentrated around quarter-ends, tax deadlines, and significant Treasury issuance dates (see Appendix C). Additionally, we calibrate the shock reversion intensity to reflect an expected shock duration of one day ($\lambda' = 1$ per day), consistent with the typical observation that these shocks fully revert by the subsequent day. We set ϕ to 6% in line with existing leverage ratio regulations and the time discount rate ρ to 2%. The steady-state deposit share α^s is calibrated externally to 0.74, the sample mean of the empirical deposit share—households' core bank deposits relative to their total money-like holdings of core deposits and money market fund (MMF) shares—over our sample (see Appendix C). Together, we denote the set of calibrated parameters as:

$$\Psi^c \equiv (\rho, \phi, \lambda, \lambda', \alpha^s).$$

[26]This analysis abstracts from additional costs of increasing the central bank balance sheet, such as, for instance, the increase in rollover risk for the consolidated government as studied by d'Avernas, de Fraisse, Ning, and Vandeweyer (2024).

Relative to the model section, we allow the transaction cost on Treasuries to depend on the outstanding supply:

$$\nu(b_t^\tau) = \nu_0 \left(b_t^\tau / \bar{b}^\tau\right)^{\gamma_\nu},$$

where b_t^τ is the outstanding stock of Treasury bonds (equal to τ_t^h in equilibrium) and $\bar{b}^\tau$ its sample mean. The economic motivation is that, as the stock of Treasuries grows relative to dealers' balance-sheet capacity, the marginal cost of intermediating an additional trade rises (Duffie, 2020; Duffie, Fleming, Keane, Nelson, Shachar, and Van Tassel, 2023); a positive elasticity γ_ν corresponds to a per-dollar trading cost that increases with supply. The level parameter ν_0 is the transaction cost evaluated at the reference (sample-mean) supply. Given the calibrated parameters and the observed series for $\underline{m}_t$ and b_t^τ, we estimate the set of parameters:

$$\Psi^e \equiv (\beta, \kappa, \nu_0, \gamma_\nu, \sigma),$$

where β scales households' liquidity-service utility, κ is the intraday-liquidity parameter, ν_0 and γ_ν govern the level and supply-elasticity of the per-trade transaction cost on Treasuries, and σ is the standard deviation of the measurement error on the observed basis.

The likelihood is constructed from the measurement equation:

$$\text{basis}_t = r^\delta(\underline{m}_t, b_t^\tau; \Psi^c, \Psi^e) - r^p(\underline{m}_t, b_t^\tau; \Psi^c, \Psi^e) + \varepsilon_t, \qquad \varepsilon_t \sim \mathcal{N}(0, \sigma^2).$$

To obtain the model-implied spread $r^\delta(\underline{m}_t, b_t^\tau; \Psi^c, \Psi^e) - r^p(\underline{m}_t, b_t^\tau; \Psi^c, \Psi^e)$, we solve the model at each date t by setting the model parameters $\underline{m}$ and b^τ equal to the observed time series for central bank reserves and Treasuries outstanding, evaluating the transaction cost at the observed Treasury supply, $\nu(b_t^\tau)$, while fixing the preference shock at its steady-state value $\alpha_t = \alpha^s$. The target basis series is the 5-year tenor Treasury cash–futures basis constructed as in Barth and Kahn (2025), smoothed with a 90-day centered moving average and expressed in basis points; the sample runs from January 2017 through February 2025. The daily time series for Treasuries (notes and bonds) and reserves outstanding are retrieved from official statements (see Appendix C for additional details). We optimize the likelihood function over the set of estimated parameters Ψ^e, conditional on $\underline{m}_t$ and b_t^τ.

The estimated parameters, detailed in Table 1, yield economically plausible and internally consistent values, indicating that the model can successfully account for the observed Treasury cash–futures basis. Notably, the estimated one-way transaction cost at the reference supply level, $\nu_0 = 1.302$ basis points, aligns with typical Treasury market transaction costs, and the estimated elasticity $\gamma_\nu = 2.466$ implies that this cost rises appreciably as the outstanding stock of Treasuries grows relative to dealers' intermediation capacity. The intraday-liquidity parameter is $\kappa = 1.455$: because reserves recirculate within the day—each dollar settling repo more than once as borrowers' payments are redeposited—a bank can sustain a repo book that exceeds its reserve stock. The model microfounds $\kappa = N(1-\bar{\varepsilon})$ as the product of intraday reserve velocity N and the

Calibrated Parameters

Parameter		Value	Unit
Time discount rate	ρ	2	percent per year
Shock arrival intensity	λ	0.069	events per day
Shock reversion arrival intensity	λ'	1	events per day
Leverage ratio regulation	ϕ	6	percent
Steady-state deposit share	α^s	0.740	–
Reference Treasury supply (sample mean)	$\bar{b}^\tau$	23.5	trillion USD

Estimated Parameters

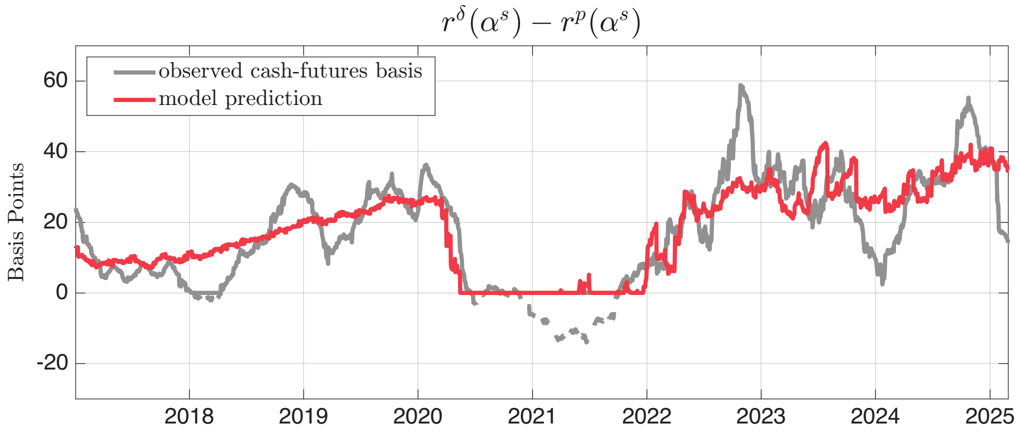
Parameter		Value	Unit
Intraday liquidity parameter	κ	1.455	–
Transaction cost level (at $b^\tau = \bar{b}^\tau$)	ν_0	1.302	basis points
Transaction-cost elasticity	γ_ν	2.466	–
Scale of liquidity-service utility	β	998	trillion USD per year
Measurement error volatility	σ	7.76	basis points

Table 1: This table reports the value of every model parameter used in our baseline quantitative analysis for both calibrated and estimated parameters, along with their units.

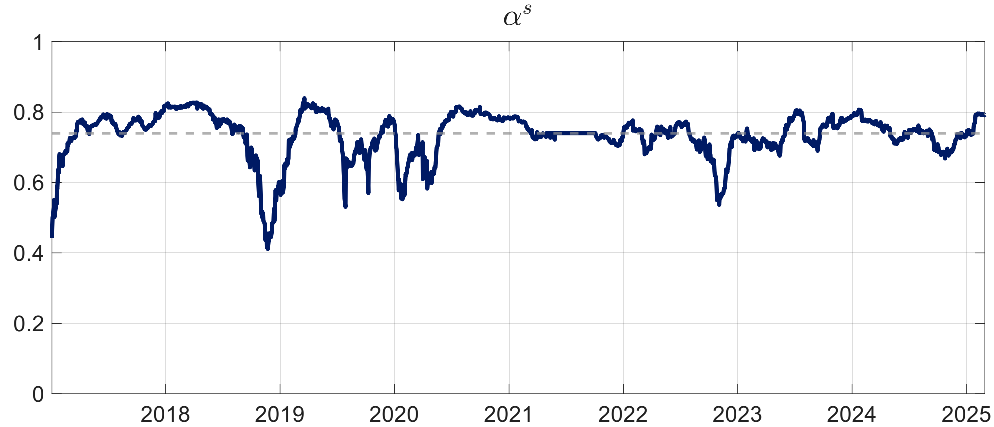
per-round non-overdraft buffer $1-\bar{\varepsilon}$; the estimate corresponds to roughly two settlement rounds with a worst-case per-round drain near 27%. The parameter $\beta = 998$ (expressed in trillion USD per year) scales liquidity utility to match the average level of the cash-futures basis. Lastly, the model's residual dispersion, $\sigma = 7.76$ basis points, captures the imperfect fit to observed data; it is roughly half the standard deviation of the observed basis itself (15.7 basis points). A residual of this magnitude is consistent with the intrinsic difficulty of measuring the cash–futures basis as a clean funding spread: it jumps when the cheapest-to-deliver security switches and embeds the time-varying value of the futures short's delivery options—sources of day-to-day volatility that motivate our 90-day moving-average target and that a structural model of funding conditions is not designed to capture.

3.2 Quantitative Implications

Empirical Fit Figure 8, Panel A, compares the model-implied Treasury cash–futures basis series with the observed data. The estimated model closely tracks the dynamics of the basis, attaining an R^2 of 0.71 on the targeted series, and captures a significant portion of the variation driven by balance-sheet conditions. During the temporary exemption of Treasuries and reserves from the supplementary leverage ratio (May 2020 – March 2021), the leverage-ratio channel is switched off in the model and the predicted basis falls to zero; the observed basis indeed turns negative over this window. This empirical fit supports the model's central mechanism:



Panel A: Observed and model-predicted Treasury cash–futures basis. Model prediction shows the model simulation with the parameters reported in Table 1. The observed Treasury cash–futures basis is the 5-year tenor series constructed as in Barth and Kahn (2025), smoothed with a 90-day centered moving average; dashed segments plot the observed series where it is negative. Treasuries and reserves were temporarily exempted from the supplementary leverage ratio from May 2020 to March 2021.



Panel B: Basis-implied deposit share α^s. The figure shows the time series of the per-date deposit share α_t^s that would allow the model to exactly match the observed Treasury cash–futures basis, to be compared with the calibrated constant $\alpha^s = 0.74$ (dashed line).

Figure 8: Empirical fit and basis-implied deposit share.

fluctuations in the Treasury cash–futures basis arise primarily from the size of the central bank's balance sheet vis-à-vis the outstanding supply of Treasuries. The result is also robust to relaxing the quasi-stationary treatment of the supply processes: in the rational-expectations extension of Internet Appendix IA.1, in which agents internalize the empirical regime dynamics of reserves and Treasury supply, the predicted basis changes by at most 5.2×10^{-5} basis points across the ten empirical supply regimes—four orders of magnitude below the $\approx$0.5 basis-point precision implied by the 90-day moving-average filter on the data target.

The model further indicates that a substantial part of the equilibrium basis compensates hedge funds for anticipated funding losses during periods when intraday liquidity constraints bind, consistent with Proposition 4. Residual variation unexplained by the model may reflect genuine abnormal profits or losses from these trades or other influences omitted from our framework.

(a) Comovements: data vs. model

	Daily		Quarterly	
	Data	Model	Data	Model
corr(basis, m)	-0.15	-0.14	-0.16	-0.14
corr(basis, b^τ)	0.45	0.55	0.47	0.53
corr(P(spike), m)	-0.73	-0.84	-0.75	-0.85
corr(HF repo, basis)	—	—	0.53	0.64

(b) Model series vs. data

	Correlation	
	Daily	Quarterly
Cash–futures basis†	0.85	0.91
Repo-spike probability	0.72	0.75
Shadow-bank repo	—	0.79

Table 2: Untargeted moments. None of these correlations is targeted; the estimation fits the cash–futures basis alone. Panel (a), *Comovements: data vs. model*, reports the correlation of the cash–futures basis with reserves m and with Treasury supply b^τ, of the repo-spike probability P(spike) with reserves, and of hedge-fund (HF) repo with the basis, each computed in the data and in the model. Panel (b), *Model series vs. data*, reports the correlation of each model-implied series with its empirical counterpart, with the cash–futures basis (†the estimation target, $R^2 = 0.71$) shown for reference. In the data, P(spike) is the 60-day frequency of days on which the general-collateral repo rate exceeds the IORB by more than 5 bp; in the model it is the index $\lambda\,(1-\alpha_t^F)$. HF repo is gross repo from SEC Form PF (via the OFR Hedge Fund Monitor) and is available only quarterly. The sample is January 2017–February 2025 (2,027 trading days; 32 quarters, 2017–2024, three-month averages).

For example, we impose in our estimation that the steady-state relative preference of households between deposits and repos, α^s, remains fixed at its calibrated value. In practice, this deposit share is subject to slow-moving variations as the opportunity costs of holding deposits evolve over time (Xiao, 2020).

Figure 8, Panel B, plots the implied path of α^s that would be required for the model to fully match the observed basis. The basis-implied series fluctuates around the calibrated constant of 0.74, consistent with the calibration capturing the central tendency of households' deposit demand over the sample; its episodic deviations line up with documented deposit-channel dynamics—flows between bank deposits and money-market instruments as deposit rates lag policy rates. In particular, to replicate the sharp increase in the basis observed at the end of 2022, α^s would need to dip to roughly 0.54 before reverting toward its calibrated level within a few months—a period of pronounced outflows from bank deposits into money market funds as policy rates rose and deposit pass-through remained limited. Additional time-varying factors affecting the supply and demand for Treasuries may also contribute to this discrepancy. Notably, the model's explanatory power diminishes during the yield curve inversion from July 2022 to August 2023 (U.S. Department of the Treasury, 2025). The corresponding pronounced uptick in the observed basis—larger than predicted by our model—could reflect an unmodeled decline in Treasury demand from carry traders, leaving hedge funds with increased residual positions. Furthermore, gradual adjustments in futures margin requirements and fluctuations in risk appetite may also play roles not captured by the constant-parameter specification. Anecdotal evidence supports this interpretation, as hedge funds reportedly reduced their leverage in cash–futures trades following the March 2020 Treasury market turmoil.[27]

[27] See, for instance, (Board of Governors of the Federal Reserve System, 2023).

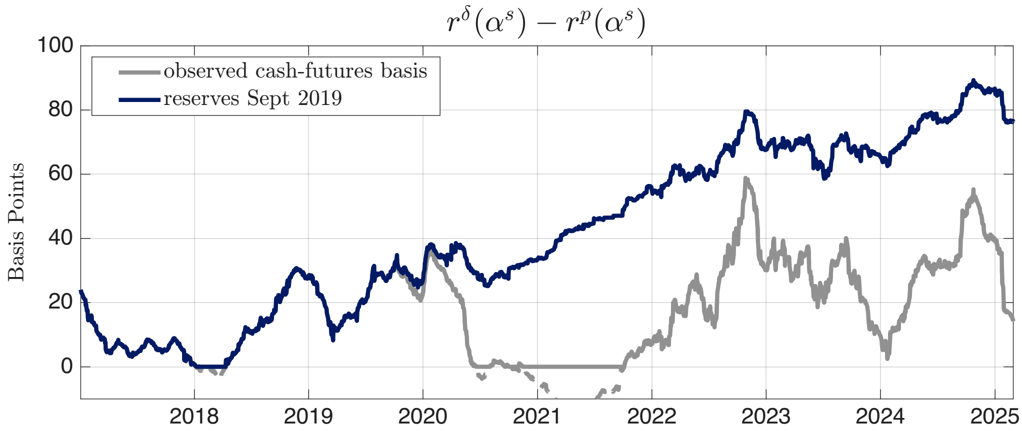
Untargeted Validation The estimation targets the cash–futures basis alone, yet the model reproduces features of the data it was not asked to fit. Panel (a) of Table 2 reports comovements in the data and in the model: in both, the basis falls with reserves and rises with Treasury supply, the repo-spike probability falls with reserves, and hedge-fund repo rises with the basis, with daily and quarterly correlations of the same sign and comparable magnitude. Panel (b) provides the more direct test, correlating each model-implied series with its empirical counterpart: the model's repo-spike probability $\lambda\,(1-\alpha_t^F)$ tracks the empirical spike frequency at 0.72 and shadow banks' repo borrowing tracks hedge funds' Form PF repo at 0.79—both untargeted, against the targeted basis's 0.85.

Fire-Sale Thresholds and Shock Persistence Besides its implication for the cash-futures basis, our estimated parameters also imply a quantitative figure for the threshold necessary to trigger entry into the fire-sale region. Plugging in our parameter estimates in Equation (33), our model implies that fire sales would occur only if repo rates were to spike above approximately 46 percentage points above the interest on reserves. Such a large threshold for fire-sales is a direct implication of Proposition 4, combined with our calibration of shock persistence $\lambda' = 1$ (shocks revert within one day).[28] Under a highly transient shock environment, hedge funds prefer to endure large temporary funding losses rather than incur the fixed transaction costs associated with liquidating Treasuries, making fire-sale episodes rare.

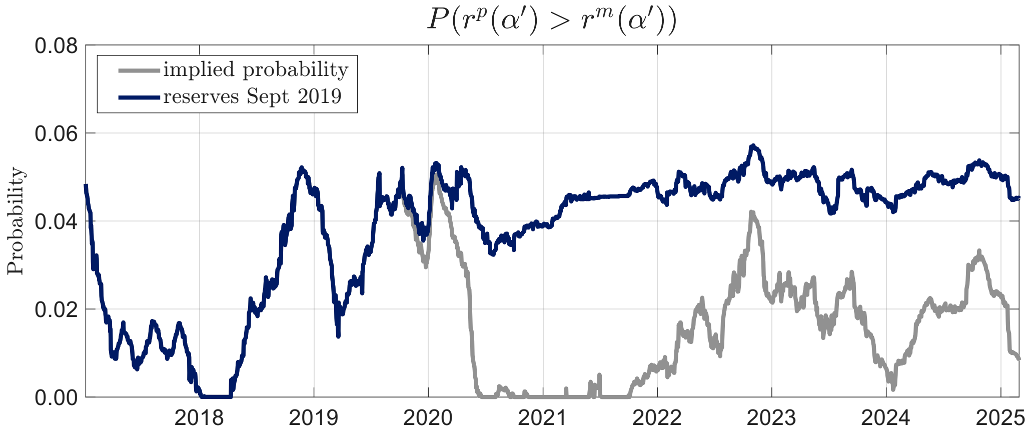
This quantitative threshold also aligns with the evidence from episodes such as the September 2019 repo spike, when the general-collateral repo rate rose to nearly 400 basis points above the interest on reserve balances (Figure 1) without triggering widespread Treasury liquidations. In contrast, the relatively moderate 76 bps repo spread above the interest on reserves observed in March 2020 coincided with sizable Treasury sales by hedge funds. According to our model, such liquidations would occur if market participants perceived the shock to persist at least 48 days—consistent with evidence of market participants' perception of a structural and persistent funding shock associated with the COVID-19 crisis—unlike the transient shocks typically associated with tax deadlines or quarter-end balance-sheet constraints.[29]

[28]The calculation implied by the model for that repo spread is given by $(\rho + 2 \times \lambda' \times 360) \times 2 \times \nu(\bar{b}^\tau) \times (1+\kappa)$, with the transaction cost evaluated at the sample-mean Treasury supply, $\nu(\bar{b}^\tau) = \nu_0$.

[29]The calculation implied by the model for that duration in days is given by $1/((0.0076/(2 \times \nu(b_t^\tau) \times (1+\kappa)) - \rho)/2/360)$, where $\nu(b_t^\tau) = 1.023$ basis points is the transaction cost evaluated at the Treasury supply of March 16, 2020. We note that this figure might overstate the physical persistence of shocks for two reasons. First, the estimate is obtained under a risk-neutral measure, whereas risk premia were large during spring 2020 (Gormsen and Koijen, 2020), so the risk-neutral persistence likely exceeds the physical one. Second, data on actual hedge fund repo borrowing rates are not available and can only be approximated by inter-dealer rates, which serves as a lower-bound for those rates. Higher observed repo rates would imply a shorter expected shock duration, as the model interprets them as indicating greater tolerance for elevated funding costs before the fire-sale threshold is reached.



Panel A: Counterfactual Treasury cash–futures basis. In this counterfactual, we set the level of reserves to remain constant from mid-September 2019 onward. The panel plots the counterfactual model-implied Treasury cash-futures basis alongside the observed basis; dashed segments plot the observed series where it is negative, as in Figure 8.



Panel B: Counterfactual probability of repo spikes. In this counterfactual, we set the level of reserves to remain constant from mid-September 2019 onward. The panel plots the counterfactual model-implied probability that the bilateral repo rate exceeds the interest rate on reserves, alongside the baseline model-implied probability.

Figure 9: Reduced central bank balance sheet counterfactual.

3.3 Counterfactual Analysis

Finally, we use the estimated model to evaluate how fiscal and monetary balance-sheet policies influence the cash–futures basis and market fragility under two counterfactual scenarios. In both exercises, we hold the deposit share at its basis-implied path α_t^s from Figure 8, Panel B, rather than at the calibrated constant, so that the counterfactual environment differs from the observed one only through the supply path; as a result, the counterfactual basis coincides with the observed series wherever the counterfactual and actual supply paths coincide.

Counterfactual 1: Smaller central bank balance sheet. We first construct a counterfactual scenario in which the central bank's balance sheet is held fixed at its mid-September 2019 level—its lowest up to that point in the sample—just before the large repo spike that prompted the Federal Reserve to conduct a series of operations to increase the supply of reserves.

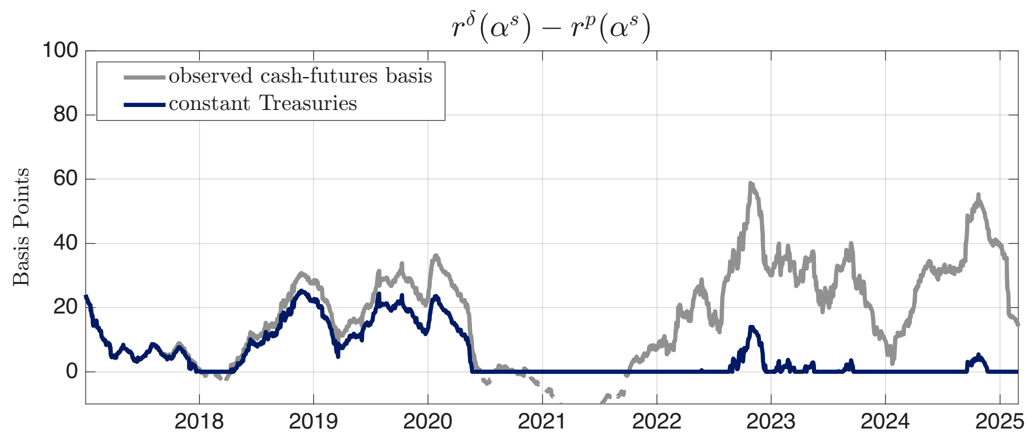
This initial response was followed by a large-scale expansion associated with the quantitative-easing programs implemented during the COVID-19 pandemic, and later by a gradual normalization. This hypothetical reduction in reserves tightens banks' intraday liquidity (IL) constraints, causing repo markets to reach the constrained region more frequently. Consequently, the model predicts repo spikes that are both larger and occur more often, an increased likelihood of fire-sale episodes, and a persistently higher Treasury cash–futures basis. As illustrated in Figure 9, absent the substantial expansion of the Fed's balance sheet in 2020, our model implies that the cash–futures basis would have remained elevated—hovering around 30 basis points through 2020 and climbing to roughly 55 basis points by the end of 2021, when the observed basis had only just turned positive—and would have continued to rise thereafter, peaking near 90 basis points toward the end of 2024, compared with an observed peak of about 60 basis points.

Counterfactual 2: No Treasury supply expansion. We next consider a second counterfactual scenario in which the Treasury's outstanding debt is held fixed at its level on the first day of the sample (January 3, 2017), rather than increasing, first gradually and then sharply following the onset of the COVID-19 pandemic. In this exercise, we let reserves supply follow its actual path. With a lower supply of Treasuries to be financed through repo, shadow-bank demand for funding declines, easing banks' IL constraints and reducing both the mean and volatility of the cash-futures basis. This effect is illustrated in Figure 10, Panel A: with reduced Treasury supply the basis would have remained close to zero for most of the post-2020 sample, with the exception of a brief increase at the end of 2022. As further illustrated in Panel B, the corresponding repo spike probabilities would also have significantly been reduced.

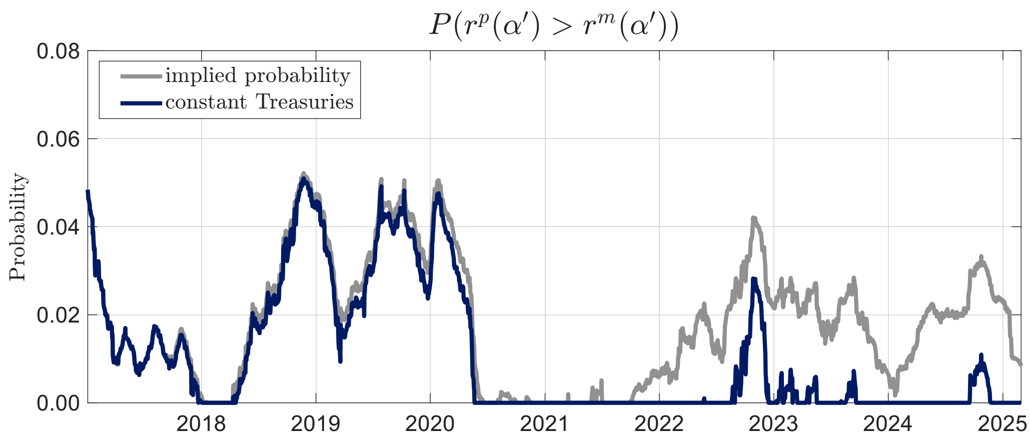
4 Conclusion

This article proposes a dynamic model to jointly explain recent disruptions observed in repo and Treasury markets. It emphasizes the central bank's balance sheet, expectations of shocks, and regulatory frictions in driving Treasury market fragility and the cash-futures basis. To maintain tractability, the framework excludes additional mechanisms that may also influence Treasury market stability, such as dealer balance-sheet capacity, margin calls, or imperfect competition. Likewise, to focus on the positive implications, it does not incorporate the broader costs of policy intervention. We leave a comprehensive normative analysis of optimal policy intervention that fully incorporates these trade-offs to future research.

A concrete policy implication of our framework is the reserve-supply rule derived from the intraday liquidity constraint: maintaining reserves above the threshold $b_t^{\tau}/(1+\kappa)$—a fixed fraction of the outstanding Treasury supply—keeps the constraint slack in every state and eliminates repo rate spikes, with selective open-market operations and the Standing Repo Facility absorbing departures from this benchmark. Beyond its microstructure rationale, this reserve-supply rule has broader implications for the monetary–fiscal interaction. By tying the central bank's



Panel A: Counterfactual Treasury cash–futures basis. In this counterfactual, the quantity of outstanding Treasuries is held fixed at its level at the beginning of the sample. The panel plots the counterfactual model-implied Treasury cash–futures basis alongside the observed basis; dashed segments plot the observed series where it is negative, as in Figure 8.



Panel B: Counterfactual probability of repo spikes. In this counterfactual, the quantity of outstanding Treasuries is held fixed at its level at the beginning of the sample. The panel plots the counterfactual model-implied probability that the bilateral repo rate exceeds the interest rate on reserves, alongside the baseline model-implied probability.

Figure 10: Reduced Treasury supply counterfactual.

balance sheet to the Treasury supply, the rule introduces an implicit accommodation of fiscal policy by monetary policy: persistent growth in fiscal issuance requires a parallel expansion of the central bank's balance sheet. A sustained accommodative stance of this form blurs the conventional distinction between technical liquidity management and the more contested role of central-bank balance-sheet operations in fiscal financing—a configuration often described as fiscal dominance. A full assessment of this regime-change risk lies outside our framework—it requires embedding the microstructure mechanism within a model with active price determination and fiscal financing—and we leave it to future research.

Overall, the paper provides theoretical foundations for understanding Treasury market fragility arising from money-market disruptions, with implications for regulatory design and the implementation of monetary policy frameworks.

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Appendices

A Solving the Model

Notation and State Space Given our assumption on the law of motion of α_t, the functional form of the transaction cost, and aggregate wealth dynamics, equilibrium prices are only a function of α_t. In the following proofs, we rewrite agents' problems in recursive form and drop the time subscript for ease of notation. We denote by $f(\cdot)$ the distribution of α' given the arrival of a Poisson shock from the steady state α^s.

First, we guess and verify that the value functions have the following form:

$$V(b;\alpha) = \xi(\alpha) + \theta(\alpha)b, \tag{34}$$

$$\overline{V}(\bar{b};\alpha) = \bar{\xi}(\alpha) + \bar{\theta}(\alpha)\bar{b}, \tag{35}$$

$$V^h(n^h;\alpha) = \xi^h(\alpha) + n^h. \tag{36}$$

Shadow Banks We can write the HJB for shadow banks as

$$\begin{aligned}\overline{V}(\bar{b}_{\text{-}};\alpha) = \max_{\bar{b}\geq 0}\Big\{&\bar{b}_{\text{-}}(r^\delta(\alpha) - r^p(\alpha))dt + (1-\rho dt)(1-\lambda(\alpha)dt)\mathbb{E}_t\big[\overline{V}(\bar{b};\alpha+d\alpha)|dN=0\big]\\ &+ (1-\rho dt)\lambda(\alpha)dt\mathbb{E}\big[\overline{V}(\bar{b};\alpha+d\alpha)|dN=1\big]\Big\}.\end{aligned} \tag{37}$$

Using Ito's lemma, the law of motion for α, and the law of motion for $\overline{\pi}$, we can rewrite the HJB in equation (37) as

$$\begin{aligned}(\rho+\lambda(\alpha))\overline{V}(\bar{b}(\alpha);\alpha) =&\bar{b}(\alpha)(r^\delta(\alpha) - r^p(\alpha))\\ &+\lambda(\alpha)\int\left(\overline{V}(\bar{b}(\alpha');\alpha') - \nu|\bar{b}(\alpha') - \bar{b}(\alpha)|\right)f(\alpha')d\alpha'.\end{aligned} \tag{38}$$

Substituting with the guess for $\overline{V}$ obtains

$$\begin{aligned}(\rho+\lambda(\alpha))\overline{V}(\bar{b}(\alpha);\alpha) =&\bar{b}(\alpha)(r^\delta(\alpha) - r^p(\alpha))\\ &+\lambda(\alpha)\int\left(\bar{\xi}(\alpha') + \bar{\theta}(\alpha')\bar{b}(\alpha') - \nu|\bar{b}(\alpha') - \bar{b}(\alpha)|\right)f(\alpha')d\alpha'.\end{aligned} \tag{39}$$

As banks can adjust their holdings of Treasuries instantaneously by paying the transaction cost, the value function given $\bar{b}_{\text{-}}$ must be equal to the value that would obtain by changing the Treasury bond holdings to the optimum, which we denote $\bar{b}^\star(\bar{b}_{\text{-}};\alpha)$:

$$\overline{V}(\bar{b}_{\text{-}};\alpha) = \max_{\bar{b}}\left\{\ \overline{V}(\bar{b};\alpha) - \nu|\bar{b} - \bar{b}_{\text{-}}|\right\} \tag{40}$$

$$= \overline{V}(\bar{b}^\star(\bar{b}_{\text{-}};\alpha);\alpha) - \nu|\bar{b}^\star(\bar{b}_{\text{-}};\alpha) - \bar{b}_{\text{-}}|. \tag{41}$$

For ease of notation, we sometimes use the short notation $\bar{b}^\star \equiv \bar{b}^\star(\bar{b}_{\text{-}};\alpha)$. Thus, $\bar{b}^\star$ is determined by

$$\overline{V}_b(\bar{b}^\star;\alpha) = \nu\,\text{sign}(\bar{b}^\star - \bar{b}_{\text{-}}) \quad \text{if } \bar{b}^\star \neq -\bar{b}_{\text{-}}, \tag{42}$$

where we use the notation $f_x = \partial f / \partial x$ for partial derivatives. Substituting for the guess for $\overline{V}$, we get

$$\overline{\theta}(\alpha) = \nu \operatorname{sign}(\overline{b}^\star - \overline{b}_-) \quad \text{if } \overline{b}^\star \neq \overline{b}_-. \tag{43}$$

Then, we can write the optimal weight on Treasuries as follows:

$$\overline{b}^\star(\overline{b}_-;\alpha) = \begin{cases} 0 & \text{if } \; \overline{\theta}(\alpha) < -\nu \\ [0, \overline{b}_-] & \text{if } \; \overline{\theta}(\alpha) = -\nu, \\ \overline{b}_- & \text{if } \; -\nu < \overline{\theta}(\alpha) < \nu, \\ [\overline{b}_-, \infty] & \text{if } \; \overline{\theta}(\alpha) = \nu. \end{cases} \tag{44}$$

We omit the case for $\overline{\theta}(\alpha) > \nu$, as this would lead to an infinite holding of Treasuries, which is infeasible in equilibrium.

Using the envelope theorem with respect to $\overline{b}$, we get

$$\begin{aligned}(\rho + \lambda(\alpha))\overline{\theta}(\alpha) =& r^\delta(\alpha) - r^p(\alpha) \\ &+ \lambda(\alpha)\frac{\partial}{\partial \overline{b}(\alpha)} \int \left(\overline{\theta}(\alpha')\overline{b}(\alpha') - \nu|\overline{b}(\alpha') - \overline{b}(\alpha)|\right) f(\alpha')d\alpha'.\end{aligned} \tag{45}$$

For $\alpha = \alpha^s$, this becomes

$$\begin{aligned}(\rho + \lambda)\overline{\theta}(\alpha^s) =& r^\delta(\alpha^s) - r^p(\alpha^s) \\ &+ \lambda \int_{\overline{b}(\alpha') \neq \overline{b}(\alpha^s)} \nu \operatorname{sign}(\overline{b}(\alpha') - \overline{b}(\alpha^s)) f(\alpha')d\alpha' \\ &+ \lambda \int_{\overline{b}(\alpha') = \overline{b}(\alpha^s)} \overline{\theta}(\alpha') f(\alpha')d\alpha'.\end{aligned} \tag{46}$$

For $\alpha \neq \alpha^s$, since after a Poisson shock we return to the steady state α^s, we get

$$\begin{aligned}(\rho + \lambda')\overline{\theta}(\alpha) =& r^\delta(\alpha) - r^p(\alpha) \\ &+ \lambda'\nu \operatorname{sign}(\overline{b}(\alpha^s) - \overline{b}(\alpha))\mathbb{1}\{\overline{b}(\alpha^s) \neq \overline{b}(\alpha)\} \\ &+ \lambda'\overline{\theta}(\alpha^s)\mathbb{1}\{\overline{b}(\alpha^s) = \overline{b}(\alpha)\}.\end{aligned} \tag{47}$$

Traditional Banks Similarly, we can write the HJB for traditional banks as

$$\begin{aligned}V(b_-;\alpha) = \max_{b \geq 0, m \geq 0, p, x \geq 0, d \geq 0, e \geq 0} \Big\{ & \mu(\alpha)dt \\ &+ (1 - \rho dt)(1 - \lambda(\alpha)dt)\mathbb{E}_t\big[V(b;\alpha + d\alpha)|dN = 0\big] \\ &+ (1 - \rho dt)\lambda(\alpha)dt\mathbb{E}\big[V(b;\alpha + d\alpha)|dN = 1\big]\Big\}\end{aligned} \tag{48}$$

where

$$\begin{aligned}\mu(\alpha) \equiv& b_- r^\delta(\alpha) + mr^m(\alpha) + pr^p(\alpha) + x(r^p(\alpha) - r^{pt}(\alpha)) \\ &- dr^d(\alpha) - er^e(\alpha)\end{aligned} \tag{49}$$

and such that $b + m + p = d + e$, $p \leq \kappa m$, and $e \geq \phi\ell$, where $\ell \equiv e + d + x - \min\{0, p\}$.

As before, the value function given b_{-} must be equal to the value that would obtain by changing the Treasury bond holdings to the optimum, which we denote $b^{\star}(b_{-};\alpha)$. For ease of notation, we sometimes use the short notation $b^{\star} \equiv b^{\star}(b_{-};\alpha)$. That is,

$$V(b_{-};\alpha) = \max_{b\geq 0} \{\, V(b;\alpha) - \nu|b - b_{-}|\} \tag{50}$$

$$= V(b^{\star};\alpha) - \nu|b^{\star} - b_{-}|. \tag{51}$$

Thus, substituting for the guess for V, we get

$$\theta(\alpha) = \nu \,\mathrm{sign}(b^{\star} - b_{-}) \quad \text{if } b^{\star} \neq b_{-}. \tag{52}$$

Thus,

$$b^{\star}(b_{-};\alpha) = \begin{cases} 0 & \text{if} \quad \theta(\alpha) < -\nu \\ [0, b_{-}] & \text{if} \quad \theta(\alpha) = -\nu, \\ b_{-} & \text{if} \quad -\nu < \theta(\alpha) < \nu, \\ [b_{-}, \infty] & \text{if} \quad \theta(\alpha) = \nu. \end{cases} \tag{53}$$

Using the same steps as for the shadow banks and substitute with the guess for V obtains

$$\begin{aligned}(\rho + \lambda(\alpha))V(b(\alpha);\alpha) = \mu(\alpha) \\ + \lambda(\alpha) \int \left(\xi(\alpha') + \theta(\alpha')b(\alpha') - \nu|b(\alpha') - b(\alpha)|\right) f(\alpha')d\alpha' \\ + \vartheta^{m}(\alpha)(\kappa m(\alpha) - p(\alpha)) + \vartheta^{e}(\alpha)(e(\alpha) - \phi\ell(\alpha)),\end{aligned} \tag{54}$$

where $\vartheta^{m}(\alpha)$ is the Lagrange multiplier on the constraint $\kappa m(\alpha) \geq p(\alpha)$ and $\vartheta^{e}(\alpha)$ is the Lagrange multiplier on the constraint $e(\alpha) \geq \phi\ell(\alpha)$. Thus, given that in equilibrium $d > 0$, the first-order condition for m, p, x, and e are given by

$$r^{m}(\alpha) - r^{d}(\alpha) = \phi\vartheta^{e}(\alpha) - \kappa\vartheta^{m}(\alpha), \tag{55}$$

$$r^{p}(\alpha) - r^{d}(\alpha) = \begin{cases} \phi\vartheta^{e}(\alpha) + \vartheta^{m}(\alpha) & \text{if } p(\alpha) > 0, \\ 0 & \text{if } p(\alpha) < 0. \end{cases} \tag{56}$$

$$r^{p}(\alpha) - r^{pt}(\alpha) = \phi\vartheta^{e}(\alpha), \tag{57}$$

$$r^{e}(\alpha) - r^{d}(\alpha) = \vartheta^{e}(\alpha). \tag{58}$$

Thus,

$$p(\alpha) = \begin{cases} \in (-\infty, 0] & \text{if } r^{p}(\alpha) - r^{d}(\alpha) = 0 \\ 0 & \text{if } 0 < r^{p}(\alpha) - r^{d}(\alpha) < \phi\vartheta^{e}(\alpha) \\ \in [0, \kappa m(\alpha)] & \text{if } r^{p}(\alpha) - r^{d}(\alpha) = \phi\vartheta^{e}(\alpha) \\ = \kappa m(\alpha) & \text{if } r^{p}(\alpha) - r^{d}(\alpha) > \phi\vartheta^{e}(\alpha). \end{cases} \tag{59}$$

As before, the envelope theorem yields

$$\begin{aligned}(\rho + \lambda(\alpha))\theta(\alpha) = & r^{\delta}(\alpha) - r^{d}(\alpha) - \phi\vartheta^{e}(\alpha) \\ & + \lambda(\alpha)\frac{\partial}{\partial b(\alpha)} \int \left(\theta(\alpha')b(\alpha') - \nu|b(\alpha') - b(\alpha)|\right) f(\alpha')d\alpha'.\end{aligned} \tag{60}$$

For $\alpha = \alpha^s$, this becomes

$$(\rho + \lambda)\theta(\alpha^s) = r^\delta(\alpha^s) - r^d(\alpha^s) - \phi\vartheta^e(\alpha^s)$$
$$+ \lambda \int_{b(\alpha') \neq b(\alpha^s)} \nu \,\mathrm{sign}(b(\alpha') - b(\alpha^s)) f(\alpha') d\alpha'$$
$$+ \lambda \int_{b(\alpha') = b(\alpha^s)} \theta(\alpha') f(\alpha') d\alpha'. \tag{61}$$

For $\alpha \neq \alpha^s$, since after a Poisson shock we return to the steady state α^s, we get

$$(\rho + \lambda')\theta(\alpha) = r^\delta(\alpha) - r^d(\alpha) - \phi\vartheta^e(\alpha)$$
$$+ \lambda'\nu \,\mathrm{sign}(b(\alpha^s) - b(\alpha))\mathbb{1}\{b(\alpha^s) \neq b(\alpha)\}$$
$$+ \lambda'\theta(\alpha^s)\mathbb{1}\{b(\alpha^s) = b(\alpha)\}. \tag{62}$$

Households Similarly, we can write the HJB for households as

$$V^h(n^h; \alpha) = \max_{c^h, d^h \geq 0, p^h \geq 0, e^h \geq 0} \Big\{ dc^h(\alpha)dt + \beta \log(h(d^h, p^h; \alpha))dt \tag{63}$$
$$+ (1 - \rho dt)(1 - \lambda(\alpha)dt)\mathbb{E}_t\big[V^h(n^h + dn^h; \alpha + d\alpha)|dN = 0\big]$$
$$+ (1 - \rho dt)\lambda(\alpha)dt\mathbb{E}\big[V^h(n^h + dn^h; \alpha + d\alpha)|dN = 1\big]\Big\}$$

where

$$h(d^h, p^h; \alpha) = (d^h)^\alpha (p^h)^{1-\alpha}, \tag{64}$$

given the law of motion for net worth

$$dn^h = (e^h r^e(\alpha) + d^h r^d(\alpha) + p^h r^{pt}(\alpha) + (b^\tau - \underline{b})r^\delta(\alpha))dt \tag{65}$$
$$- \underline{m} r^m(\alpha)dt + d\pi(\alpha) + d\overline{\pi}(\alpha) + \nu|db(\alpha)| - dc^h(\alpha), \tag{66}$$

and such that $e^h + d^h + p^h = n^h + \tau^h$. We can rewrite the HJB as follows:

$$(\rho + \lambda(\alpha))V^h(n^h; \alpha) = c^h(\alpha) + \beta \log(h(\alpha)) + V_n^h(n^h; \alpha)dn^h/dt \tag{67}$$
$$+ \lambda(\alpha) \int V^h(n^h + dn^h; \alpha') f(\alpha') d\alpha', \tag{68}$$

The first-order conditions for households are given by

$$r^{pt}(\alpha) - r^d(\alpha) = \beta \left(\frac{\alpha}{d^h(\alpha)} - \frac{1 - \alpha}{p^h(\alpha)} \right), \tag{69}$$
$$r^e(\alpha) - r^d(\alpha) = \beta \frac{\alpha}{d^h(\alpha)}, \tag{70}$$

Together with the traditional bank first-order condition for equities, it implies that the LR constraint is always binding. The envelope condition for net worth yields

$$r^e(\alpha) = \rho. \tag{71}$$

Also note that given the aggregate balance sheet and budget constraints, in equilibrium $n_t^h = c_t^h = 0$.

B Proofs of Section 2

State Space Partitioning We define four disjoint sets of equilibria that correspond to different dynamics in the pricing of the bilateral repo. Our analysis below characterizes how shocks to the state variable α are shifting equilibrium across those sets.

Definition 2. *Let $\mathcal{A}$ be the set of* **arbitraged** *repo market equilibria, defined as $\{\mathcal{M}(\alpha) \in \mathcal{A} \mid p(\alpha) < 0\}$.*

Definition 3. *Let $\mathcal{S}$ be the set of* **segmented** *repo market states, defined as $\{\mathcal{M}(\alpha) \in \mathcal{S} \mid p(\alpha) = 0$ and $r^p(\alpha) < r^m(\alpha)\}$.*

Definition 4. *Let $\mathcal{U}$ be the set of* **unconstrained** *repo market states, defined as $\{\mathcal{M}(\alpha) \in \mathcal{U} \mid r^p(\alpha) = r^m(\alpha)\}$.*

Definition 5. *Let $\mathcal{C}$ be the set of* **constrained** *repo market states, defined as $\{\mathcal{M}(\alpha) \in \mathcal{C} \mid r^p(\alpha) > r^m(\alpha)\ \}$.*

Definition 6. *Let $\mathcal{F}$ be the set of* **fire-sale** *states, a subset of constrained repo market states $\mathcal{C}$, defined as $\{\mathcal{M}(\alpha) \in \mathcal{F} \mid b(\alpha) \neq b(\alpha^s)\}$.*

We define as *arbitraged* the set of equilibria in which traditional banks are borrowing in bilateral repos; as *segmented* the ones in which traditional banks are not marginal in bilateral repos due to the balance sheet cost; as *unconstrained* equilibria in which the IL constraint is not binding for traditional banks and bilateral repo rates equal to the interest on reserves; as *constrained* the ones in which traditional banks are constrained by the IL regulation, and where bank repo rates are above the interest on reserves; and as *fire sale*, the ones in which traditional banks adjust their Treasury bonds holdings when entering the shock state.

Intermediate Results We begin with a set of intermediate results before providing the proofs of our propositions. Below, we take stock of the assumptions made in the main text.

Assumption 1. *There is always a state in which shadow banks fire-sell Treasuries to traditional banks: $\exists \alpha' \in (\underline{\alpha}, 1), \overline{b}(\alpha') < \overline{b}(\alpha^s)$.*

Assumption 2. *In the steady state, traditional banks hold some Treasuries: $b(\alpha^s) > 0$.*

Assumption 3. *The traditional bank dealer subsidiary has a positive balance sheet size: $x(\alpha) > 0$.*

Assumption 4. *The household balance sheet is larger than the quantity of repo available from traditional banks: $\tau^h > \kappa m$.*

Lemma A1. *If there exists α' such that $0 < b(\alpha^s) < b(\alpha')$, then $\theta(\alpha^s) = -\overline{\theta}(\alpha^s) = -\nu$, $\theta(\alpha') = -\overline{\theta}(\alpha') = \nu$, $\overline{b}(\alpha^s) > \overline{b}(\alpha')$, and there does not exist a shock state α' such that $b(\alpha') < b(\alpha^s)$ or $\overline{b}(\alpha^s) < \overline{b}(\alpha')$.*

Proof. From the optimality condition for b in equation (53), we have that for the traditional bank to be incentivized to increase its holding of Treasuries and pay the adjustment cost when moving from α^s to α' and vice versa, then it must be that $\theta(\alpha') = \nu$ and $\theta(\alpha^s) \leq -\nu$. Thus, there cannot exist another state such that $b(\alpha') < b(\alpha^s)$ as this would require $\theta(\alpha^s) = \nu$.

The market-clearing condition for the Treasury market is given by

$$b(\alpha) + \overline{b}(\alpha) + \underline{b} = b^{\tau} \quad \forall\alpha. \tag{72}$$

Thus, the reverse must be true for $\overline{b}(\alpha)$ and $\overline{\theta}(\alpha)$. In addition, given that $b(\alpha^s) > 0$ in Assumption 2, we have $\theta(\alpha^s) = -\nu$. This further implies that

$$r^p(\alpha') - r^d(\alpha') = \phi\vartheta^e(\alpha') + (\rho + \lambda')(\nu - \overline{\theta}(\alpha')) + 2\lambda'\nu \tag{73}$$

from the envelope theorem for traditional and shadow banks. Given $\overline{\theta}(\alpha') \leq -\nu$, $r^p(\alpha') - r^d(\alpha') > \phi\vartheta^e(\alpha')$. By the traditional bank first-order conditions, we have $p(\alpha') > 0$. Combining the triparty and bilateral repo market clearing conditions, we have $\overline{p}(\alpha') = p^h(\alpha') + p(\alpha') > 0$. Hence, $\overline{\theta}(\alpha') = -\nu$. □

Lemma A1 pins down $\overline{\theta}(\alpha^s) = -\theta(\alpha^s) = \nu$ in the steady state and $\overline{\theta}(\alpha') = -\theta(\alpha') = -\nu$ in the fire sale region under Assumption 1. Below, we formalize what happens in each state.

Steady State: $\alpha = \alpha^s$. Given Lemma A1, the envelope theorem for traditional and shadow banks leads to the following expressions for rates relative to $r^d(\alpha^s)$:

$$\begin{aligned} r^{\delta}(\alpha^s) - r^d(\alpha^s) =& \phi\vartheta^e(\alpha^s) - (\rho + \lambda)\nu \\ &- \lambda \int_{\mathcal{M}(\alpha')\in\mathcal{F}} \nu f(\alpha')d\alpha' - \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} \theta(\alpha')f(\alpha')d\alpha', \end{aligned} \tag{74}$$

$$r^{\delta}(\alpha^s) - r^p(\alpha^s) = (\rho + \lambda)\nu + \lambda \int_{\mathcal{M}(\alpha')\in\mathcal{F}} \nu f(\alpha')d\alpha' - \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} \overline{\theta}(\alpha')f(\alpha')d\alpha'. \tag{75}$$

Thus,

$$\begin{aligned} r^p(\alpha^s) - r^d(\alpha^s) =& \phi\vartheta^e(\alpha^s) - 2(\rho + \lambda)\nu \\ &- 2\lambda \int_{\mathcal{M}(\alpha')\in\mathcal{F}} \nu f(\alpha')d\alpha' + \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} (\overline{\theta}(\alpha') - \theta(\alpha'))f(\alpha')d\alpha'. \end{aligned} \tag{76}$$

Thus, $0 \leq r^p(\alpha^s) - r^d(\alpha^s) < \phi\vartheta^e(\alpha^s)$ as $\overline{\theta}(\alpha') - \theta(\alpha') \leq 2\nu$. This further implies that $p(\alpha^s) \leq 0$ from the first-order condition for p. Hence, $\vartheta^m(\alpha^s) = 0$ and the IL constraint is not binding in the steady state. From the combined envelope conditions of traditional and shadow banks, we get

$$\begin{aligned} r^{pt}(\alpha^s) - r^d(\alpha^s) =& -2(\rho + \lambda)\nu \\ &- 2\lambda \int_{\mathcal{M}(\alpha')\in\mathcal{F}} \nu f(\alpha')d\alpha' + \lambda \int_{\mathcal{M}(\alpha')\notin\mathcal{F}} (\overline{\theta}(\alpha') - \theta(\alpha'))f(\alpha')d\alpha'. \end{aligned} \tag{77}$$

For ease of notation, define $\Theta^s \equiv r^{pt}(\alpha^s) - r^d(\alpha^s)$. We have two cases.

- Case $p(\alpha^s) < 0$, thus $\mathcal{M}(\alpha^s) \in \mathcal{A}$. Then $\Theta^s = r^{pt}(\alpha^s) - r^d(\alpha^s) = -\phi\beta\frac{\alpha^s}{d^h(\alpha^s)}$. Together with $r^{pt}(\alpha^s) - r^d(\alpha^s) = \beta\left(\frac{\alpha^s}{d^h(\alpha^s)} - \frac{1-\alpha^s}{p^h(\alpha^s)}\right)$, we get $d^h(\alpha^s) = -\beta\frac{\phi\alpha^s}{\Theta^s}$ and $p^h(\alpha^s) =$

$-\beta\frac{1-\alpha^s}{\Theta^s}\frac{\phi}{1+\phi}$. With the budget constraint $\tau^h = e^h(\alpha^s) + p^h(\alpha^s) + d^h(\alpha^s)$ and the LR constraint $e(\alpha^s) = \phi\ell(\alpha^s) = \phi(e(\alpha^s) + d(\alpha^s) - p(\alpha^s) + x(\alpha^s)) = \phi(\tau^h - p(\alpha^s))$, we can solve for $p(\alpha^s) = \tau^h(1 - 1/\phi) - \frac{\beta}{\Theta^s}\left(\alpha^s + \frac{1-\alpha^s}{1+\phi}\right)$.

We can also write Θ^s as a function of $\overline{p}(\alpha^s)$, which will be useful later:

$$\overline{p}(\alpha^s) = p^h(\alpha^s) + p(\alpha^s) = -\beta\frac{1-\alpha^s}{\Theta^s}\frac{\phi}{1+\phi} + \tau^h(1 - 1/\phi) - \frac{\beta}{\Theta^s}\left(\alpha^s + \frac{1-\alpha^s}{1+\phi}\right). \tag{78}$$

With further algebra, we get

$$\Theta^s = \frac{\beta}{\tau^h(1 - 1/\phi) - \overline{p}(\alpha^s)}. \tag{79}$$

- Case $p(\alpha^s) = 0$, thus $\mathcal{M}(\alpha^s) \in \mathcal{S}$. With the budget constraint $\tau^h = p^h(\alpha^s) + d^h(\alpha^s) + e^h(\alpha^s)$, and the LR constraint $e(\alpha^s) = \phi\ell(\alpha^s) = \phi\tau^h$, we can solve for $d^h(\alpha^s) = \tau^h(1 - \phi) - p^h(\alpha^s)$. For ease of notation, we define

$$\mathbf{p}^h(s,\alpha) \equiv \begin{cases} \frac{s\tau^h(1-\phi)-\beta+\mathcal{G}(s,\alpha)}{2s} & \text{if } s \neq 0, \\ \tau^h(1-\phi)(1-\alpha) & \text{if } s = 0. \end{cases} \tag{80}$$

where

$$\mathcal{G}(s,\alpha) \equiv \sqrt{\beta^2 + s^2(\tau^h)^2(1-\phi)^2 + 2\beta s\tau^h(1-\phi)(1-2\alpha)}. \tag{81}$$

Then $\mathbf{p}^h(r^{pt} - r^d, \alpha)$ is the solution[30] of Equation (26) for p^h in terms of a spread $r^{pt} - r^d$ and a liquidity preference parameter α: $p^h(\alpha^s) = \mathbf{p}^h(\Theta^s, \alpha^s)$.

Given $p^h(\alpha^s) = \overline{p}(\alpha^s)$, it is also useful to write

$$\Theta^s = \beta\left(\frac{\alpha^s}{\tau^h(1-\phi) - \overline{p}(\alpha^s)} - \frac{1-\alpha^s}{\overline{p}(\alpha^s)}\right). \tag{82}$$

Then, combined with the shadow bank balance sheet constraint, repo markets clearing conditions, and the Treasury bond market-clearing condition, we get

$$\overline{b}(\alpha^s) = p^h(\alpha^s) + p(\alpha^s) \tag{83}$$

and

$$b(\alpha^s) = b^\tau - \underline{b} - p^h(\alpha^s) - p(\alpha^s). \tag{84}$$

Shock States: $\alpha' \in (\underline{\alpha}, 1)$**.** Given Lemma A1, the envelope theorem for traditional and shadow banks leads to the following expressions for rates relative to $r^\delta(\alpha')$:

$$(\rho + \lambda')\theta(\alpha') = r^\delta(\alpha') - r^d(\alpha') - \phi\vartheta^e(\alpha') - \lambda'\nu, \tag{85}$$
$$(\rho + \lambda')\overline{\theta}(\alpha') = r^\delta(\alpha') - r^p(\alpha') + \lambda'\nu, \tag{86}$$

[30] Only this root is relevant, since the other root implies that either p^h or d^h is negative.

and

$$(\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) = r^p(\alpha') - r^d(\alpha') - \phi\vartheta^e(\alpha') - 2\lambda'\nu. \tag{87}$$

From the combined envelope conditions of traditional and shadow banks, we get

$$r^{pt}(\alpha') - r^d(\alpha') = (\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu. \tag{88}$$

For ease of notation, define $\Theta' \equiv r^{pt}(\alpha') - r^d(\alpha')$. Since $\theta(\alpha') - \overline{\theta}(\alpha') \leq 2\nu$, we have $\Theta' \leq 2\rho\nu + 4\lambda'\nu$, and $\Theta' = 2\rho\nu + 4\lambda'\nu$ when there is a fire sale. There is no discontinuity with respect to α' in the range of possible values for Θ'.

States with the IL Constraint Not Binding: $\alpha'|\vartheta^m(\alpha') = 0$. Consider some shock $\alpha' \in (\underline{\alpha}, 1)$ such that the IL constraint does not bind. From the traditional bank first-order conditions, we get $\Theta' = r^{pt}(\alpha') - r^d(\alpha') \leq 0$. Hence, it is impossible that $\theta(\alpha') = -\overline{\theta}(\alpha') = \nu$. By Lemma A1, there is no fire sale: $b(\alpha') = b(\alpha^s)$ and $\overline{b}(\alpha') = \overline{b}(\alpha^s)$. We get

$$\overline{p}(\alpha') = \overline{p}(\alpha^s) = p^h(\alpha') + p(\alpha') = p^h(\alpha^s) + p(\alpha^s). \tag{89}$$

We have three cases.

- Case $p(\alpha') < 0$. Thus, $\mathcal{M}(\alpha') \in \mathcal{A}$. Then $\Theta' = r^{pt}(\alpha') - r^d(\alpha') = -\phi\beta\frac{\alpha'}{d^h(\alpha')}$. Together with $r^{pt}(\alpha') - r^d(\alpha') = \beta\left(\frac{\alpha'}{d^h(\alpha')} - \frac{1-\alpha'}{p^h(\alpha')}\right)$, we get $d^h(\alpha') = -\beta\frac{\phi\alpha'}{\Theta'}$ and $p^h(\alpha') = -\beta\frac{1-\alpha'}{\Theta'}\frac{\phi}{1+\phi}$. With the budget constraint $\tau^h = p^h(\alpha') + d^h(\alpha') + e^h(\alpha')$ and the LR constraint $e(\alpha') = \phi\ell(\alpha') = \phi(\tau^h - p(\alpha'))$, we can solve for $p(\alpha') = \tau^h(1 - 1/\phi) - \frac{\beta}{\Theta'}\left(\alpha' + \frac{1-\alpha'}{1+\phi}\right)$.

  We can then pin down Θ' using (89):

  $$p^h(\alpha^s) + p(\alpha^s) = -\beta\frac{1-\alpha'}{\Theta'}\frac{\phi}{1+\phi} + \tau^h(1 - 1/\phi) - \frac{\beta}{\Theta'}\left(\alpha' + \frac{1-\alpha'}{1+\phi}\right). \tag{90}$$

  With further algebra, we get

  $$\Theta' = \frac{\beta}{\tau^h(1 - 1/\phi) - \overline{p}(\alpha^s)}. \tag{91}$$

- Case $\Theta' < 0$ and $p(\alpha') \geq 0$. Then $r^{pt}(\alpha') < r^d(\alpha')$ and $p(\alpha') = 0$ by traditional bank first-order conditions. Thus, $\mathcal{M}(\alpha') \in \mathcal{S}$.

  Then, $p^h(\alpha') = \overline{p}(\alpha^s)$, $d^h(\alpha') = \tau^h(1 - \phi) - \overline{p}(\alpha^s)$, and

  $$\Theta' = \beta\left(\frac{\alpha'}{\tau^h(1 - \phi) - \overline{p}(\alpha^s)} - \frac{1-\alpha'}{\overline{p}(\alpha^s)}\right). \tag{92}$$

- Case $\Theta' = 0$, thus $\mathcal{M}(\alpha') \in \mathcal{U}$. By equation (80), $p^h(\alpha') = \tau^h(1 - \phi)(1 - \alpha')$. Then $p(\alpha') = \overline{p}(\alpha^s) - p^h(\alpha')$, and $d^h(\alpha') = \tau^h(1 - \phi)\alpha'$.

States with the IL Constraint Binding: $\alpha'|\vartheta^m(\alpha') > 0$. Consider some shock $\alpha' \in (\underline{\alpha}, 1)$ such that the IL constraint binds. Then we have $p(\alpha') = \kappa m(\alpha')$ and $\vartheta^m > 0$. Therefore, from the first-order conditions of the traditional banks, it must be that $r^p(\alpha')-r^d(\alpha') > \phi\vartheta^e(\alpha')$ and $\Theta' = r^{pt}(\alpha') - r^d(\alpha') > 0$.

We first consider the case when no fire sale occurs: $\overline{b}(\alpha') = \overline{b}(\alpha^s)$. We get

$$\overline{p}(\alpha') = \overline{p}(\alpha^s) = p^h(\alpha') + p(\alpha'). \tag{93}$$

Since $p(\alpha') = \kappa\underline{m}$, $p^h(\alpha') = \overline{p}(\alpha^s) - \kappa\underline{m}$, and

$$\Theta' = \beta\left(\frac{\alpha'}{\tau^h(1-\phi) - \overline{p}(\alpha^s) + \kappa\underline{m}} - \frac{1-\alpha'}{\overline{p}(\alpha^s) - \kappa\underline{m}}\right). \tag{94}$$

If the solution gives $\Theta' > 2\nu\rho + 4\lambda'\nu$, then this contradicts with $\theta - \overline{\theta} \leq 2\nu$. A fire sale must occur. In this case, we have $\Theta' = 2\nu\rho + 4\lambda'\nu$ and we can solve for $p^h(\alpha') = \mathbf{p}^h(\Theta', \alpha')$:

Lemma A2. *If $\alpha' \in \mathcal{A}$ and $\alpha'' \in \mathcal{S}\cup\mathcal{U}\cup\mathcal{C}$ and $p^h(\alpha') \geq p^h(\alpha'')$, then $d^h(\alpha') < d^h(\alpha'')$.*

If $\alpha' \in \mathcal{S}$ and $\alpha'' \in \mathcal{U}\cup\mathcal{C}$ and $p^h(\alpha') \geq p^h(\alpha'')$, then $d^h(\alpha') \leq d^h(\alpha'')$.

If $\alpha', \alpha'' \in \mathcal{C}$ and $p^h(\alpha') \geq p^h(\alpha'')$, then $d^h(\alpha') \leq d^h(\alpha'')$.

Proof. Given $e(\alpha) = \phi\ell(\alpha) = \phi(\tau^h - \min\{0, p(\alpha)\})$, we get

$$d^h(\alpha) = \tau^h - p^h(\alpha) - e^h(\alpha) \tag{95}$$
$$= \tau^h(1-\phi) - p^h(\alpha) + \phi\min\{0, p(\alpha)\}. \tag{96}$$

Assume $\alpha' \in \mathcal{A}$ and $\alpha'' \in \mathcal{S}\cup\mathcal{U}\cup\mathcal{C}$. Then $p(\alpha') < 0$ and $p(\alpha'') \geq 0$. Thus, if $p^h(\alpha') \geq p^h(\alpha'')$, then $d^h(\alpha') < d^h(\alpha'')$.

Assume $\alpha' \in \mathcal{S}$ and $\alpha'' \in \mathcal{U}\cup\mathcal{C}$. Then $p(\alpha') = 0$ and $p(\alpha'') \geq 0$. Thus, if $p^h(\alpha') \geq p^h(\alpha'')$, then $d^h(\alpha') \leq d^h(\alpha'')$.

Assume $\alpha', \alpha'' \in \mathcal{C}$. Then $p(\alpha') = p(\alpha'') = \kappa m$. Thus, if $p^h(\alpha') \geq p^h(\alpha'')$, then $d^h(\alpha') \leq d^h(\alpha'')$. ☐

Lemma A3. *If $\alpha' \geq \alpha''$, $d^h(\alpha') < d^h(\alpha'')$, and $p^h(\alpha') > p^h(\alpha'')$, then $r^{pt}(\alpha') - r^d(\alpha') > r^{pt}(\alpha'') - r^d(\alpha'')$*

Proof. Define

$$g(\alpha, d, p) \equiv \frac{\alpha}{d} - \frac{1-\alpha}{p}, \text{ where } \alpha \in (0,1) \text{ and } d \in (0, \tau^h) \text{ and } p \in (0, \tau^h). \tag{97}$$

Then $g(\alpha, d, p)$ is increasing in α and p and decreasing in d.

By the household first-order condition, $r^{pt}(\alpha)-r^d(\alpha) = \beta g(\alpha, d^h(\alpha), p^h(\alpha))$. Given that $\beta > 0$, if $\alpha' \geq \alpha''$, $d^h(\alpha') < d^h(\alpha'')$, and $p^h(\alpha') > p^h(\alpha'')$, then $r^{pt}(\alpha') - r^d(\alpha') > r^{pt}(\alpha'') - r^d(\alpha'')$. ☐

Lemma A4. *$\mathcal{A}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{C}$ are intervals. Furthermore, $\mathcal{A}$, $\mathcal{S}$, $\mathcal{U}$, and $\mathcal{C}$ form a partition of $(\alpha^s, 1)$ and $\mathcal{F}$ is an interval subset of $\mathcal{C}$. Finally, we can write*

$$\forall\mathcal{M}(\alpha') \in \mathcal{A}, \mathcal{M}(\alpha'') \in \mathcal{S}\cup\mathcal{U}\cup\mathcal{C} : \alpha' < \alpha''; \tag{98}$$
$$\forall\mathcal{M}(\alpha') \in \mathcal{S}, \mathcal{M}(\alpha'') \in \mathcal{U}\cup\mathcal{C} : \alpha' < \alpha''; \tag{99}$$
$$\forall\mathcal{M}(\alpha') \in \mathcal{U}, \mathcal{M}(\alpha'') \in \mathcal{C} : \alpha' < \alpha''; \tag{100}$$
$$\forall\mathcal{M}(\alpha') \in \mathcal{C}\setminus\mathcal{F}, \mathcal{M}(\alpha'') \in \mathcal{F} : \alpha' \leq \alpha''. \tag{101}$$

Proof. We first take a look at the rate spread between the triparty repo and deposits.

Next, we prove every statement sequentially.

- Statement 98. Consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{A}$ and $\mathcal{M}(\alpha'') \in \mathcal{S}$. By definition, we have that $p(\alpha') < 0$ and $p(\alpha'') = 0$. Since there are no fire sales in $\mathcal{A}$ and $\mathcal{S}$, $\overline{p}(\alpha') = \overline{p}(\alpha'') = \overline{p}(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $p^h(\alpha') > p^h(\alpha'')$. Thus, by Lemma A2, $d^h(\alpha') < d^h(\alpha'')$.

  Assume by way of contradiction that $\alpha' \geq \alpha''$. Then, by Lemma A3, $\Theta' > \Theta''$. From the first-order conditions of traditional banks and households, $\Theta' = -\phi\vartheta^e(\alpha') = -\phi\beta\alpha'/d^h(\alpha')$ and $\Theta'' \geq -\phi\vartheta^e(\alpha'') = -\phi\beta\alpha''/d^h(\alpha'')$. Thus, it must be that $\alpha'/d^h(\alpha') \leq \alpha''/d^h(\alpha'')$ and $d^h(\alpha') \geq d^h(\alpha'')$ a contradiction with the above statement. Therefore, it must be that $\alpha' < \alpha''$.

- Statement 99. Consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{S}$ and $\mathcal{M}(\alpha'') \in \mathcal{U}$. By definition, we have that $p(\alpha') = 0$ and $p(\alpha'') \geq 0$. Since there are no fire sales in $\mathcal{A}$ and $\mathcal{S}$, $\overline{p}(\alpha') = \overline{p}(\alpha'') = \overline{p}(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $p^h(\alpha') \geq p^h(\alpha'')$. Thus, by Lemma A2, $d^h(\alpha') \leq d^h(\alpha'')$.

  Assume by way of contradiction that $\alpha' \geq \alpha''$. Then, by Lemma A3, $\Theta' \geq \Theta''$. From the first-order conditions of traditional banks and households, $\Theta' < 0$ and $\Theta'' = 0$, contradiction with the above statement. Therefore, it must be that $\alpha' < \alpha''$ and $\Theta' < \Theta''$.

- Statement 100. Next, consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{U}$ and $\mathcal{M}(\alpha'') \in \mathcal{C} \setminus \mathcal{F}$. We have that $p(\alpha') \in [0, \kappa m]$ and $p(\alpha'') = \kappa m$ by the traditional bank first-order condition. Since there are no fire sales in $\mathcal{U}$ and $\mathcal{C} \setminus \mathcal{F}$, $\overline{p}(\alpha') = \overline{p}(\alpha'') = \overline{p}(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $p^h(\alpha'') \leq p^h(\alpha')$. Thus, by Lemma A2, $d^h(\alpha') < d^h(\alpha'')$.

  Assume by way of contradiction that $\alpha' \geq \alpha''$. Then, by Lemma A3, $\Theta' \geq \Theta''$. From the first-order conditions of traditional banks and households, $\Theta' = 0$ and $\Theta'' > 0$, a contradiction with the above statement. Therefore, it must be that $\alpha' < \alpha''$ and $\Theta' < \Theta''$.

- Statement 101. Finally, consider arbitrary $\mathcal{M}(\alpha') \in \mathcal{C} \setminus \mathcal{F}$ and $\mathcal{M}(\alpha'') \in \mathcal{F}$. We have that $p(\alpha') = p(\alpha'') = \kappa m$. Given that there is a fire-sale in $\mathcal{F}$, $\overline{p}(\alpha') = \overline{p}(\alpha^s) > \overline{p}(\alpha'')$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $p^h(\alpha'') < p^h(\alpha')$. Thus, by Lemma A2, $d^h(\alpha') < d^h(\alpha'')$.

  Assume by way of contradiction that $\alpha'' < \alpha'$. Then, by Lemma A3, $\Theta' > \Theta''$.

  Given Equation (87), $\Theta' \leq 2\rho\nu + 4\lambda'\nu$ and $\Theta'' = 2\rho\nu + 4\lambda'\nu$. Thus, $\Theta'' \leq \Theta'$, a contradiction with the above statement. Therefore, it must be that $\alpha' \leq \alpha''$ and $\Theta' \leq \Theta''$.

□

Let us define the thresholds[31] of each partition as follows:

$$\mathcal{A} = [\underline{\alpha}, \alpha^S), \quad \mathcal{S} = [\alpha^S, \alpha^U), \; \mathcal{U} = [\alpha^U, \alpha^{IL}], \; \mathcal{C} = (\alpha^{IL}, 1), \; \mathcal{F} = (\alpha^F, 1) \tag{102}$$

and $\alpha^S < \alpha^U < \alpha^{IL} \leq \alpha^F$. We next show that traditional banks are not short repo in equilibrium.

[31] Note that if $\underline{\alpha} > \alpha^S$, then $\mathcal{A} = \emptyset$ and $\mathcal{S} = [\underline{\alpha}, \alpha^U)$, if $\underline{\alpha} > \alpha^U$, then $\mathcal{A} = \mathcal{S} = \emptyset$ and $\mathcal{U} = [\underline{\alpha}, \alpha^{IL})$, and so forth.

Lemma A5. *It is necessary that $\alpha^s < \alpha^S \leq \underline{\alpha}$ and $\mathcal{A} = \emptyset$.*

Proof. From Equations (77) and (88), we have

$$\Theta^s \leq -2\nu\rho - 4\lambda\nu\mathbb{P}(\mathcal{M}(\alpha) \in \mathcal{F}) \tag{103}$$

and for any $\alpha' \in [\underline{\alpha}, 1)$,

$$\Theta' = (\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) + 2\lambda'\nu \geq -2\nu\rho \tag{104}$$

given $-2\nu \leq \overline{\theta}(\alpha') - \theta(\alpha') \leq 2\nu$. Thus, $\Theta^s < \Theta'$ for any α'.

Assume $p(\alpha^s) \leq p(\alpha')$. Since there is no fire sale, $\overline{p}(\alpha') = \overline{p}(\alpha^s)$. Thus, combining the bilateral and triparty repo market-clearing conditions, we must have $p^h(\alpha^s) \geq p^h(\alpha')$. Thus, by Lemma A2, $d^h(\alpha^s) \leq d^h(\alpha')$.

Assume by way of contradiction that $\alpha^s \geq \alpha'$. Then, by Lemma A3, $\Theta^s \geq \Theta'$, a contradiction. Therefore, since $p(\alpha^s) \leq 0$, $\alpha^s < \alpha^S$.

Since we assume $p(\alpha^s) < 0$, given Equations (79) and (90), $\Theta^s = \frac{\beta}{\tau^h(1-1/\phi)-\overline{p}(\alpha^s)} = \Theta'$ for $\alpha' \in \mathcal{A}$, which contradicts $\Theta^s < \Theta'$. Thus, it must be that $\alpha^S \leq \underline{\alpha}$ and $\mathcal{A} = \emptyset$.

□

Lemma A6. *The thresholds solve for*

$$\alpha^S = \frac{\tau^h(1-\phi) - \overline{p}(\alpha^s)}{\tau^h(1-\phi) + \phi\overline{p}(\alpha^s)} \tag{105}$$

$$\alpha^U = \frac{\tau^h(1-\phi) - \overline{p}(\alpha^s)}{\tau^h(1-\phi)} \tag{106}$$

$$\alpha^{IL} = \frac{\tau^h(1-\phi) - \overline{p}(\alpha^s) + \kappa m}{\tau^h(1-\phi)} \tag{107}$$

$$\alpha^F = \alpha^{IL}\left(1 + \frac{\Theta^F}{\beta}\left(\overline{p}(\alpha^s) - \kappa m\right)\right) \tag{108}$$

where $\Theta^F = 2\nu(\rho + \lambda') + 2\lambda'\nu$.

Proof. This is quickly verified by solving the traditional banks and household's first-order condition at the partition boundaries. □

Proposition 1. *The equilibrium is unique in prices $\{r^m, r^p, r^{pt}, r^d, r^e\}$ and quantities $\{b, f, m, p, x, d, e, \overline{b}, \overline{f}, \overline{p}, e^h, d^h, p^h\}$ as functions of $\alpha \in [\underline{\alpha}, 1)$. Also, the cash-futures implied risk-free rate is bounded by $\underline{r}^\delta(\alpha) \leq r^\delta(\alpha) \leq \overline{r}^\delta(\alpha)$, where*

$$\underline{r}^\delta(\alpha') \equiv \max\{r^d(\alpha') + \phi\vartheta^e(\alpha') - \rho\nu, r^p(\alpha') - (\rho + 2\lambda')\nu\}$$
$$\overline{r}^\delta(\alpha') \equiv \min\{r^d(\alpha') + \phi\vartheta^e(\alpha') + (\rho + 2\lambda')\nu, r^p(\alpha') + \rho\nu\}$$

for $\alpha' \in [\underline{\alpha}, 1)$.[32]

Proof of Proposition 1. Given Lemma A4, we get that for $\alpha' \in (\underline{\alpha}, \alpha^F]$, $\overline{b}(\alpha') = \overline{b}(\alpha^s)$ and $b(\alpha') = b(\alpha^s)$, and for $\alpha'' \in (\alpha^F, 1]$, $\overline{b}(\alpha'') < \overline{b}(\alpha^s)$ and $b(\alpha'') > b(\alpha^s)$. Thus, given $\Theta' =$

[32] See the proof in Appendix B for equivalent boundaries at $\alpha = \alpha^s$.

$(\rho+\lambda')(\theta(\alpha')-\overline{\theta}(\alpha'))+2\lambda'\nu$, we can rewrite equation (77) as

$$\Theta^s + 2\nu(\rho+\lambda) + \frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\underline{\alpha}}^{1}\Theta' d\alpha' = \frac{2\lambda\lambda'\nu}{\rho+\lambda'}. \tag{109}$$

This equation solves for $\overline{p}(\alpha^s)$, which then pins down Θ^s and Θ' for all α' given the derivations above. Because Θ^s is continuously differentiable with respect to $\overline{p}(\alpha^s)$ and differentiable with respect to α', we can use Leibniz' rule and obtain

$$\frac{\partial}{\partial\overline{p}(\alpha^s)}\int_{\underline{\alpha}}^{1}\Theta' d\alpha' = \int_{\underline{\alpha}}^{1}\frac{\partial\Theta'}{\partial\overline{p}(\alpha^s)}d\alpha'. \tag{110}$$

Using the derivations of Θ' above, we get

$$\frac{\partial\Theta'}{\partial\overline{p}(\alpha^s)} = \begin{cases} \beta\left(\frac{\alpha'}{(\tau^h(1-\phi)-\overline{p}(\alpha^s))^2}+\frac{1-\alpha'}{\overline{p}(\alpha^s)^2}\right) & \text{if } \alpha'\in\mathcal{S}, \\ 0 & \text{if } \alpha'\in\mathcal{U}, \\ \beta\left(\frac{\alpha'}{(\tau^h(1-\phi)-\overline{p}(\alpha^s)+\kappa m)^2}+\frac{1-\alpha'}{(\overline{p}(\alpha^s)-\kappa m)^2}\right) & \text{if } \alpha'\in\mathcal{C}\backslash\mathcal{F}, \\ 0 & \text{if } \alpha'\in\mathcal{F}. \end{cases} \tag{111}$$

Similarly,

$$\frac{\partial\Theta^s}{\partial\overline{p}(\alpha^s)} > 0. \tag{112}$$

Therefore, the left-hand side of Equation (109) is monotonically increasing in $\overline{p}(\alpha^s)$ and there is a unique solution to $\overline{p}(\alpha^s)$.

We now consider the bounds on $\overline{\theta}$. Note that θ and $\overline{\theta}$ lie between $-\nu$ and ν and that

$$\theta(\alpha')-\overline{\theta}(\alpha') = \frac{\Theta'-2\lambda'\nu}{\rho+\lambda'}. \tag{113}$$

We can write the bounds of $\overline{\theta}$ as follows:

$$\overline{\theta}(\alpha')\in\left[-\nu-\frac{\Theta'-2\lambda'\nu}{\rho+\lambda'},\nu\right] \quad \text{if } \alpha'\leq\alpha^{\Theta}, \tag{114}$$

and

$$\overline{\theta}(\alpha')\in\left[-\nu,\nu-\frac{\Theta'-2\lambda'\nu}{\rho+\lambda'}\right] \quad \text{if } \alpha'>\alpha^{\Theta}, \tag{115}$$

where α^{Θ} is defined as the solution to $\Theta(\alpha^{\Theta}) = 2\lambda'\nu$. Since $0<\Theta(\alpha^{\Theta})<\Theta^F$, we have $\alpha^{\Theta}\in[\alpha^{IL},\alpha^F)$. Given that Θ' is increasing in α', it is well-defined and given by

$$2\lambda'\nu = \beta\left(\frac{\alpha^{\Theta}}{\tau^h(1-\phi)-\overline{p}(\alpha^s)+\kappa\underline{m}}-\frac{1-\alpha^{\Theta}}{\overline{p}(\alpha^s)-\kappa\underline{m}}\right). \tag{116}$$

From the shadow bank envelope theorem, we get

$$r^{\delta}(\alpha') = r^p(\alpha')+(\rho+\lambda')\overline{\theta}(\alpha')-\lambda'\nu. \tag{117}$$

Using the definition of θ and $\overline{\theta}$ and first-order conditions, we get that the interest rate on

Treasuries is bounded by $\underline{r}^{\delta}(\alpha) \leq r^{\delta}(\alpha) \leq \overline{r}^{\delta}(\alpha)$, where

$$\underline{r}^{\delta}(\alpha) \equiv \max\{r^{d}(\alpha) + \phi\vartheta^{e}(\alpha) - \rho\nu, r^{p}(\alpha) - (\rho + 2\lambda')\nu\}, \tag{118}$$

and

$$\overline{r}^{\delta}(\alpha) \equiv \min\{r^{d}(\alpha) + \phi\vartheta^{e}(\alpha) + (\rho + 2\lambda')\nu, r^{p}(\alpha) + \rho\nu\}. \tag{119}$$

for $\alpha \in [\underline{\alpha}, 1)$. When $\mathcal{M}(\alpha) \in \mathcal{F}$, the cash-futures basis is pinned down by $r^{p}(\alpha) - r^{\delta}(\alpha) = (\rho + 2\lambda')\nu$.

The envelope theorem gives

$$r^{\delta}(\alpha^{s}) - r^{p}(\alpha^{s}) = (\rho + \lambda)\nu - \frac{\lambda}{1 - \underline{\alpha}} \int_{\underline{\alpha}}^{1} \overline{\theta}(\alpha')d\alpha' \tag{120}$$

We can then integrate over $\min\{\overline{\theta}(\alpha')\}$ and $\max\{\overline{\theta}(\alpha')\}$ to get the bounds for the spreads in the steady state.

□

Lemma A7. *The IL constraint binds ($\vartheta_t^m > 0$) if and only if $b^{\tau} - \underline{b} - b_t > (1 - \alpha_t)\tau^{h}(1 - \phi) + \kappa m$.*

Proof. Assume that $b^{\tau} - \underline{b} - b_t > (1 - \alpha_t)\tau^{h}(1 - \phi) + \kappa m$. Combining the market-clearing conditions, we get $p_t^h = b^{\tau} - \underline{b} - b_t - p_t$. Thus, $p_t^h > (1 - \alpha_t)\tau^{h}(1 - \phi) + \kappa m - p_t \geq (1 - \alpha_t)\tau^{h}(1 - \phi)$. Then, by the household first-order conditions, $r_t^{pt} > r_t^d$ and by the traditional bank first-order conditions, $\vartheta_t^m > 0$.

If $\vartheta_t^m > 0$, then, $p_t = \kappa m > 0$ and by the traditional bank first-order conditions, $r_t^{pt} > r_t^d$. Thus, by the household first-order condition, $p_t^h > (1 - \alpha_t)\tau^{h}(1 - \phi)$. Combining the market-clearing conditions, we get $b^{\tau} - \underline{b} - b_t - \kappa m = p_t^h > (1 - \alpha_t)\tau^{h}(1 - \phi)$. □

Lemma 1. *$r^{p}(\alpha') > r^{m}(\alpha')$ if and only if $\alpha' > \alpha^{IL}$, where α^{IL} is such that*

$$\underbrace{b^{\tau} - \underline{b} - b(\alpha^{IL})}_{\text{repo demand}} = \underbrace{(1 - \alpha^{IL})\tau^{h}(1 - \phi) + \kappa \underline{m}}_{\text{maximum repo capacity}}. \tag{32}$$

Proof of Lemma 1. If $\vartheta_t^m > 0$, then $p_t = \kappa m > 0$. Thus, by the traditional bank first-order conditions, $r_t^p > r_t^m$. If $r_t^p > r_t^m$, by the traditional bank first-order conditions, $\vartheta_t^m > 0$.

Thus, given Lemma A7, $r_t^p > r_t^m$ if and only if $b^{\tau} - \underline{b} - b_t > (1 - \alpha_t)\tau^{h}(1 - \phi) + \kappa m$.

□

Lemma A8. *The boundary α^F is such that*

$$\frac{\partial \alpha^F}{\partial \overline{p}(\alpha^s)} < 0 \quad \text{and} \quad \frac{\partial \alpha^F}{\partial m} > 0. \tag{121}$$

Proof. Given Lemma A6

$$\frac{\partial \alpha^F}{\partial \overline{p}(\alpha^s)} \propto -1 - \frac{\Theta^F}{\beta}\left(\overline{p}(\alpha^s) - \kappa m\right) + \left(\tau^{h}(1 - \phi) - \overline{p}(\alpha^s) + \kappa m\right)\frac{\Theta^F}{\beta}. \tag{122}$$

Since

$$\Theta^F \leq \frac{\beta}{\tau^{h}(1 - \phi) - \overline{p}(\alpha^s) + \kappa m}, \tag{123}$$

then

$$\left(\tau^h(1-\phi)-\overline{p}(\alpha^s)+\kappa m\right)\frac{\Theta^F}{\beta}\leq 1 \tag{124}$$

and Equation (122) is strictly negative.

Additionally,

$$\frac{\partial\alpha^F}{\partial m}\propto 1+\frac{\Theta^F}{\beta}(\overline{p}(\alpha^s)-\kappa m)-\frac{\Theta^F}{\beta}\left(\tau^h(1-\phi)-\overline{p}(\alpha^s)+\kappa m\right)>0. \tag{125}$$

□

Lemma A9. Θ^s *is such that* $\Theta^s < -2\nu\rho$.

Proof. Given Equation (109), we know that

$$\Theta^s+2\rho\nu\frac{\rho+\lambda'+\lambda}{\rho+\lambda'}+\frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\underline{\alpha}}^{1}\Theta' d\alpha'=0. \tag{126}$$

Given Equation (113),

$$-2\nu\leq\frac{\Theta'-2\lambda'\nu}{\rho+\lambda'}\leq 2\nu \tag{127}$$

and

$$-\Theta'\leq 2\nu\rho. \tag{128}$$

Thus,

$$\Theta^s=-2\rho\nu\frac{\rho+\lambda'+\lambda}{\rho+\lambda'}-\frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\underline{\alpha}}^{1}\Theta' d\alpha' \tag{129}$$

$$<-2\rho\nu\frac{\rho+\lambda'+\lambda}{\rho+\lambda'}+\frac{\lambda}{\rho+\lambda'}2\rho\nu=-2\rho\nu. \tag{130}$$

□

Lemma A10. *We have that*

$$\frac{\partial r_t^p-r_t^m}{\partial\Theta_t}>0. \tag{131}$$

Proof. From the traditional bank's first-order conditions, we get

$$r_t^p-r_t^m=\Theta_t+\kappa\vartheta_t^m. \tag{132}$$

Thus,

$$r_t^p-r_t^m=\begin{cases}\Theta_t & \text{if } \alpha_t\notin\mathcal{C},\\ (1+\kappa)\Theta_t & \text{if } \alpha_t\in\mathcal{C}.\end{cases} \tag{133}$$

□

Proposition 2. *If* $\sqrt{\lambda/\big((1-\underline{\alpha})(\rho+\lambda')\big)} < \kappa\underline{b}/b^{\tau}$*, a tighter leverage ratio constraint* ϕ *results in larger shadow banks' Treasury holdings in the steady state, a higher probability of fire sales, and higher repo rate spikes. That is,*

$$\frac{\partial \bar{b}(\alpha^s)}{\partial \phi} > 0, \quad \frac{\partial \mathbb{P}\big(\bar{b}(\alpha') < \bar{b}(\alpha^s)\big)}{\partial \phi} > 0, \quad \textit{and} \quad \frac{\partial (r^p(\alpha') - r^m(\alpha'))}{\partial \phi} > 0.$$

Proof of Proposition 2. Throughout, reserve market clearing (condition (c)) gives $m = \underline{m}$, the central bank balance sheet gives $\underline{b} = \underline{m}$, and the Treasury identity gives $b^{\tau} = \tau^h$; hence $\kappa\underline{b}/b^{\tau} = \kappa m/\tau^h$.

Using equation (109), we define the implicit function $g(\cdot,\cdot)$ as

$$g(\overline{p}(\alpha^s),\phi) \equiv \Theta^s(\overline{p}(\alpha^s),\phi) + 2\rho\nu\frac{\rho+\lambda'+\lambda}{\rho+\lambda'} \tag{134}$$

$$+\frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\underline{\alpha}}^{1}\Theta'(\overline{p}(\alpha^s),\phi)d\alpha'. \tag{135}$$

Given $g(\overline{p}(\alpha^s),\phi)=0$, the implicit function theorem states that

$$\frac{\partial \overline{p}(\alpha^s)}{\partial \phi} = -\frac{\partial g(\overline{p}(\alpha^s),\phi)/\partial\phi}{\partial g(\overline{p}(\alpha^s),\phi)/\partial \overline{p}(\alpha^s)}. \tag{136}$$

From Section B, we know that the sign of the denominator is strictly positive. We can derive

$$\frac{\partial \Theta'}{\partial \phi} = \begin{cases} \frac{\beta\alpha'\tau^h}{(\tau^h(1-\phi)-\overline{p}(\alpha^s))^2} & \text{if } \alpha' \in \mathcal{S}, \\ 0 & \text{if } \alpha' \in \mathcal{U}, \\ \frac{\beta\alpha'\tau^h}{(\tau^h(1-\phi)-\overline{p}(\alpha^s)+\kappa m)^2} & \text{if } \alpha' \in \mathcal{C}\backslash\mathcal{F}, \\ 0 & \text{if } \alpha' \in \mathcal{F}, \end{cases} \tag{137}$$

and

$$\frac{\partial \Theta^s}{\partial \phi} = -\frac{\beta}{(\tau^h(1-1/\phi)-\overline{p}(\alpha^s))^2}\frac{\tau^h}{\phi^2}. \tag{138}$$

Taking the integral and using Lemma A6, we get

$$\frac{\partial g(\overline{p}(\alpha^s),\phi)}{\partial \phi} \leq -\frac{\beta\tau^h}{(\tau^h)^2} + \frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')}\frac{\beta\tau^h}{2}\left(\frac{1}{(\kappa m)^2}-\frac{1}{(\tau^h)^2}\right), \tag{139}$$

which is strictly negative if

$$\frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')} < \left(\frac{\kappa m}{\tau^h}\right)^2. \tag{140}$$

This gives

$$\frac{\partial \overline{p}(\alpha^s)}{\partial \phi} = \frac{\partial \bar{b}(\alpha^s)}{\partial \phi} > 0 \tag{141}$$

Given Lemma A10,

$$\frac{\partial(r^p(\alpha') - r^m(\alpha'))}{\partial\phi} = \frac{\partial(r^p(\alpha') - r^m(\alpha'))}{\partial\Theta'}\frac{\partial\Theta'}{\partial\phi} > 0. \tag{142}$$

Note that

$$\frac{d\alpha^F}{d\phi} = \frac{\partial\alpha^F}{\partial\overline{p}(\alpha^s)}\frac{\partial\overline{p}(\alpha^s)}{\partial\phi} + \frac{\partial\alpha^F}{\partial\phi} \tag{143}$$

and

$$\frac{\partial\alpha^F}{\partial\phi} = -\frac{(\overline{p}(\alpha^s) - \kappa m)(1 + \Theta^F/\beta(\overline{p}(\alpha^s) - \kappa m))}{\tau^h(1-\phi)^2} < 0. \tag{144}$$

Hence, by Lemma A8 and $\frac{\partial\overline{p}(\alpha^s)}{\partial\phi} > 0$ we have

$$\frac{d\alpha^F}{d\phi} = \frac{\partial\alpha^F}{\partial\overline{p}}\frac{\partial\overline{p}}{\partial\phi} + \frac{\partial\alpha^F}{\partial\phi} < 0. \tag{145}$$

Thus,

$$\frac{d\mathbb{P}(\bar{b}(\alpha') < \bar{b}(\alpha^s))}{\partial\phi} > 0. \tag{146}$$

$\square$

Proposition 3. *A lower shock arrival intensity λ results in larger shadow banks' Treasury holdings in the steady state, a higher probability of fire sales, and higher repo rate spreads to interest on reserves. That is,*

$$\frac{\partial\bar{b}(\alpha^s)}{\partial\lambda} < 0, \quad \frac{\partial\mathbb{P}\big(\bar{b}(\alpha') < \bar{b}(\alpha^s)\big)}{\partial\lambda} < 0, \quad \textit{and} \quad \frac{\partial(r_t^p - r_t^m)}{\partial\lambda} < 0.$$

Proof of Proposition 3. Define the implicit function $g(\cdot,\cdot)$ as

$$g(\overline{p}(\alpha^s), \lambda) \equiv \Theta^s(\overline{p}(\alpha^s)) + 2\rho\nu\frac{\rho + \lambda' + \lambda}{\rho + \lambda'} \tag{147}$$

$$+ \frac{\lambda}{(1-\underline{\alpha})(\rho + \lambda')}\int_{\underline{\alpha}}^{1}\Theta'(\overline{p}(\alpha^s))d\alpha'. \tag{148}$$

Given $g(\overline{p}(\alpha^s), \lambda) = 0$, the implicit function theorem states that

$$\frac{\partial\overline{p}(\alpha^s)}{\partial\lambda} = -\frac{\partial g(\overline{p}(\alpha^s), \lambda)/\partial\lambda}{\partial g(\overline{p}(\alpha^s), \lambda)/\partial\overline{p}(\alpha^s)}. \tag{149}$$

From Section B, we know that the sign of the denominator is strictly positive. Given lemma A9,

$$\frac{\partial g(\overline{p}(\alpha^s), \lambda)}{\partial\lambda} > 0 \tag{150}$$

Thus,

$$\frac{\partial \overline{p}(\alpha^s)}{\partial \lambda} = \frac{\partial \overline{b}(\alpha^s)}{\partial \lambda} < 0. \tag{151}$$

From Lemma A8, we have

$$\frac{\partial \mathbb{P}[\overline{b}(\alpha') < \overline{b}(\alpha^s)]}{\partial \lambda} \propto \frac{\partial (1-\alpha^F)}{\partial \lambda} = -\frac{\partial \alpha^F}{\partial \overline{p}(\alpha^s)} \frac{\partial \overline{p}(\alpha^s)}{\partial \lambda} < 0. \tag{152}$$

Given Lemma A10,

$$\frac{\partial (r^p(\alpha') - r^m(\alpha'))}{\partial \lambda} < 0. \tag{153}$$

□

Lemma A11. *We have that*

$$r_t^\delta - r_t^p = -\frac{\Theta_t}{2}. \tag{154}$$

Proof. From the shadow banks' envelope condition in Equation (45) and using the midpoint, $\overline{\theta}_t = -(\Theta_t - 2\lambda'\nu)/(\rho + \lambda')/2$, we get

$$(\rho + \lambda)\nu = r^\delta(\alpha^s) - r^p(\alpha^s) + \frac{\lambda\lambda'\nu}{\rho + \lambda'} - \frac{\lambda}{(\rho + \lambda')(1 - \underline{\alpha})} \int_{\underline{\alpha}}^1 \frac{\Theta'}{2} d\alpha' \tag{155}$$

and

$$\lambda'\nu - \frac{\Theta'}{2} = r^\delta(\alpha') - r^p(\alpha') + \lambda'\nu. \tag{156}$$

In both cases,

$$r_t^\delta - r_t^p = -\frac{\Theta_t}{2}. \tag{157}$$

□

Proposition 4. *A longer shock duration (a lower λ') results in smaller shadow banks' Treasury holdings in the steady state, lower repo spreads to interest on reserves, and a higher cash-futures basis. That is,*

$$\frac{\partial \overline{b}(\alpha^s)}{\partial \lambda'} > 0, \quad \frac{\partial (r_t^p - r_t^m)}{\partial \lambda'} > 0, \quad \textit{and} \quad \frac{\partial (r_t^\delta - r_t^p)}{\partial \lambda'} < 0.$$

Proof of Proposition 4. Define the implicit function $g(\cdot, \cdot)$ as

$$g(\overline{p}(\alpha^s), \lambda') \equiv \Theta^s(\overline{p}(\alpha^s)) + 2\nu(\rho + \lambda) - \frac{2\lambda\lambda'\nu}{\rho + \lambda'} \tag{158}$$

$$+ \frac{\lambda}{(1 - \underline{\alpha})(\rho + \lambda')} \int_{\underline{\alpha}}^1 \Theta'(\overline{p}(\alpha^s)) d\alpha'. \tag{159}$$

Given $g(\overline{p}(\alpha^s), \lambda') = 0$, the implicit function theorem states that

$$\frac{\partial \overline{p}(\alpha^s)}{\partial \lambda'} = -\frac{\partial g(\overline{p}(\alpha^s), \lambda')/\partial \lambda'}{\partial g(\overline{p}(\alpha^s), \lambda')/\partial \overline{p}(\alpha^s)}. \tag{160}$$

From Section B, we know that the sign of the denominator is strictly positive. Given $-\Theta' \leq 2\rho\nu$

$$\frac{\partial g(\overline{p}(\alpha^s), \lambda')}{\partial \lambda'} < 0 \tag{161}$$

This gives $\partial \overline{p}(\alpha^s)/\partial \lambda' = \partial \bar{b}(\alpha^s)/\partial \lambda' > 0$.

Given Lemma A10,

$$\frac{\partial (r_t^p - r_t^m)}{\partial \lambda'} > 0. \tag{162}$$

Given Lemma A11

$$\frac{\partial (r_t^\delta - r_t^p)}{\partial \lambda'} = -\frac{1}{2}\left(\frac{\partial \Theta_t}{\partial \lambda'} + \frac{\partial \Theta_t}{\partial \overline{p}(\alpha^s)}\frac{\partial \overline{p}(\alpha^s)}{\partial \lambda'}\right) < 0. \tag{163}$$

□

Proposition 5. *In the scarce-reserves regime ($\underline{m} \leq \tau^h/(1+\kappa)$, implied by Assumption 1), a smaller central bank balance sheet $\underline{b}$ results in smaller shadow banks' Treasury holdings in the steady state, a larger probability of fire sales, and a higher Treasury cash-futures basis. That is,*

$$\frac{\partial \bar{b}(\alpha^s)}{\partial \underline{b}} > 0, \quad \frac{\partial \mathbb{P}\big(\bar{b}(\alpha') < \bar{b}(\alpha^s)\big)}{\partial \underline{b}} < 0, \quad \textit{and} \quad \frac{\partial (r^\delta(\alpha^s) - r^p(\alpha^s))}{\partial \underline{b}} < 0.$$

Proof of Proposition 5. Throughout, reserve market clearing (condition (c)) gives $m = \underline{m}$ and the central bank balance sheet gives $\underline{b} = \underline{m}$, so $\partial/\partial \underline{b} = \partial/\partial m$.

Using equation (109), we define the implicit function $g(\cdot,\cdot)$ as

$$g(\overline{p}(\alpha^s), m) \equiv \Theta^s(\overline{p}(\alpha^s)) + 2\rho\nu\frac{\rho+\lambda'+\lambda}{\rho+\lambda'} \tag{164}$$

$$+ \frac{\lambda}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\underline{\alpha}}^{1} \Theta'(\overline{p}(\alpha^s), m)d\alpha'. \tag{165}$$

Given $g(\overline{p}(\alpha^s), m) = 0$, the implicit function theorem states that

$$\frac{\partial \overline{p}(\alpha^s)}{\partial m} = -\frac{\partial g(\overline{p}(\alpha^s), m)/\partial m}{\partial g(\overline{p}(\alpha^s), m)/\partial \overline{p}(\alpha^s)}. \tag{166}$$

We can derive

$$\frac{\partial \Theta'}{\partial m} = \begin{cases} 0 & \text{if } \alpha' \in \mathcal{S}\cup\mathcal{U}\cup\mathcal{F}, \\ -\kappa\frac{\partial \Theta'}{\partial \overline{p}(\alpha^s)} & \text{if } \alpha' \in \mathcal{C}\backslash\mathcal{F}, \end{cases} \tag{167}$$

Using Leibniz' rule,

$$\frac{\partial g(\overline{p}(\alpha^s), m)}{\partial m} = -\frac{\lambda\beta\kappa}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\alpha^{IL}}^{\alpha^F}\frac{\partial\Theta'}{\partial\overline{p}(\alpha^s)}d\alpha', \tag{168}$$

$$\frac{\partial g(\overline{p}(\alpha^s), m)}{\partial\overline{p}(\alpha^s)} = \frac{\beta}{(\tau^h(1-1/\phi)-\overline{p}(\alpha^s))^2} + \frac{\lambda\beta}{(1-\underline{\alpha})(\rho+\lambda')}\int_{\underline{\alpha}}^{1}\frac{\partial\Theta'}{\partial\overline{p}(\alpha^s)}d\alpha'. \tag{169}$$

Thus,

$$0 < \frac{\partial\overline{p}(\alpha^s)}{\partial m} < \kappa. \tag{170}$$

Note that

$$\frac{d\Theta_t}{dm} = \begin{cases} \frac{\partial\Theta_t}{\partial\overline{p}(\alpha^s)}\frac{\partial\overline{p}(\alpha^s)}{\partial m} > 0 & \text{if } \alpha_t \in \mathcal{S}\cup\mathcal{U}\cup\mathcal{F}, \\ \frac{\partial\Theta'}{\partial m} + \frac{\partial\Theta_t}{\partial\overline{p}(\alpha^s)}\frac{\partial\overline{p}(\alpha^s)}{\partial m} & \text{if } \alpha_t \in \mathcal{C}\backslash\mathcal{F}. \end{cases} \tag{171}$$

For $\alpha' \in \mathcal{C}\backslash\mathcal{F}$, we can further derive

$$\frac{d\Theta_t}{dm} = -\kappa\frac{\partial\Theta_t}{\partial\overline{p}(\alpha^s)} + \frac{\partial\Theta_t}{\partial\overline{p}(\alpha^s)}\frac{\partial\overline{p}(\alpha^s)}{\partial m} < 0. \tag{172}$$

Given Lemma A10

$$\frac{\partial(r_t^p - r_t^m)}{\partial m} \propto \frac{d\Theta_t}{dm}. \tag{173}$$

Given Lemma A11

$$\frac{\partial(r_t^\delta - r_t^p)}{\partial m} \propto -\frac{d\Theta_t}{dm}. \tag{174}$$

Given Lemma A8

$$\frac{\partial\alpha^F}{\partial m} = -\kappa\frac{\partial\alpha^F}{\partial\overline{p}(\alpha^s)}. \tag{175}$$

Thus,

$$\frac{d\alpha^F}{dm} = \frac{\partial\alpha^F}{\partial m} + \frac{\partial\alpha^F}{\partial\overline{p}(\alpha^s)}\frac{\partial\overline{p}(\alpha^s)}{\partial m} = -\frac{\partial\alpha^F}{\partial\overline{p}(\alpha^s)}\left(\kappa - \frac{\partial\overline{p}(\alpha^s)}{\partial m}\right) > 0. \tag{176}$$

□

The remaining results concern the abundant-reserves regime $\underline{m}(1+\kappa) > \tau^h$ of Proposition 6. In this regime Assumption 1 no longer applies—Lemma A12 below shows that no fire sales occur—and we retain only Assumptions 2 to 4 together with the restriction $\mathbb{P}(b(\alpha') > b(\alpha^s)) = 0$.

Lemma A12. *If $\underline{m}(1+\kappa) > \tau^h$, no fire sale transactions occur in equilibrium and $\mathcal{C} = \emptyset$.*

Proof. The condition $\underline{m}(1+\kappa) > \tau^h$ implies

$$\alpha^{IL} = \frac{\tau^h(1-\phi) - \overline{p}(\alpha^s) + \kappa\underline{m}}{\tau^h(1-\phi)} > 1, \tag{177}$$

so the IL constraint never binds. Using envelope conditions in the shock states,

$$\Theta' = (\rho + \lambda')\big(\theta(\alpha') - \overline{\theta}(\alpha')\big) - \lambda'\big(\theta(\alpha^s) - \overline{\theta}(\alpha^s)\big). \tag{178}$$

Assume by way of contradiction there exists $\alpha' \in [\underline{\alpha}, 1)$ with $\theta(\alpha') = \nu$ and $\overline{\theta}(\alpha') = -\nu$, implying $\theta(\alpha^s) = -\nu$ and $\overline{\theta}(\alpha^s) = \nu$. Then

$$\Theta' = (\rho + \lambda')(2\nu) + 2\lambda'\nu = 2\rho\nu + 4\lambda'\nu > 0, \tag{179}$$

which contradicts $\Theta'(\alpha') \leq 0$ for $\alpha' \notin \mathcal{C}$. $\square$

Proposition 6. *If $\underline{m} > \tau^h/(1+\kappa)$, no fire sales occur and reserves provide no further liquidity benefits; moreover, for $\underline{m}$ sufficiently large, additional units of central bank reserves strictly decrease net-of-transaction-costs liquidity benefits.*

Proof of Proposition 6. By Lemma A12, for all $\underline{m} > \tau^h/(1+\kappa)$ we have $\mathcal{C} = \emptyset$ and no fire sales occur, so additional reserves provide no further liquidity benefits through this channel. We retain the restriction $\mathbb{P}(b(\alpha') > b(\alpha^s)) = 0$. We now demonstrate that for sufficiently high $\underline{m}$, the liquidity benefits strictly decrease, establishing $\theta(\alpha^s) < -\nu$ and $b(\alpha^s) = 0$. Given Lemma A12,

$$-\phi\vartheta^e(\alpha) \leq \ \Theta(\alpha) \leq \ 0. \tag{180}$$

for all α. Combining envelope conditions gives

$$(\rho + \lambda)(\theta(\alpha^s) - \overline{\theta}(\alpha^s)) = \Theta^s + \lambda \int (\theta(\alpha') - \overline{\theta}(\alpha'))\, f(\alpha')\, d\alpha', \tag{181}$$

$$(\rho + \lambda')(\theta(\alpha') - \overline{\theta}(\alpha')) = \Theta' + \lambda'(\theta(\alpha^s) - \overline{\theta}(\alpha^s)). \tag{182}$$

Taken together, we obtain

$$\theta(\alpha^s) = \overline{\theta}(\alpha^s) + \frac{\rho + \lambda'}{\rho(\rho + \lambda + \lambda')} \left(\Theta^s + \frac{\lambda}{\rho + \lambda'} \int \Theta'\, f(\alpha')\, d\alpha' \right) \tag{183}$$

From market clearing conditions, $b(\alpha^s) + \overline{p}(\alpha^s) + \underline{m} = b^\tau$. Thus, if $b(\alpha^s) = 0$, then $\underline{m} = \tau^h - \overline{p}(\alpha^s)$ and

$$\Theta^s = -\frac{\beta\phi}{\tau^h(1-\phi) + \phi\overline{p}(\alpha^s)} = -\frac{\beta\phi}{\tau^h - \phi\underline{m}} \tag{184}$$

and

$$\Theta' = \beta \left(\frac{\alpha'}{\tau^h(1-\phi) - \overline{p}(\alpha^s)} - \frac{1-\alpha'}{\overline{p}(\alpha^s)} \right) = \beta \left(\frac{\alpha'}{\underline{m} - \phi\tau^h} - \frac{1-\alpha'}{\tau^h - \underline{m}} \right) \tag{185}$$

for $\alpha' \in \mathcal{S}$. Thus, as $\underline{m} \to \tau^h$, $\theta(\alpha^s) < -\nu$, and

$$\frac{\partial \overline{p}(\alpha^s)}{\partial \underline{m}} = -1. \tag{186}$$

In steady state, the liquidity benefits are given by

$$\log(h(d^h, p^h; \alpha^s)) = \alpha \log\left(\tau^h(1-\phi) + \phi\overline{p}(\alpha^s) - (1+\phi)p^h(\alpha^s)\right) + (1-\alpha^s)\log\left(p^h(\alpha^s)\right). \tag{187}$$

Also,

$$p^h(\alpha^s) = \frac{(1-\alpha^s)\left(\tau^h(1-\phi) + \phi\overline{p}(\alpha^s)\right)}{1+\phi}. \tag{188}$$

Thus,

$$\frac{\partial h(\alpha)}{\partial \underline{m}} \propto \left(\frac{1-\alpha}{p^h} - \frac{\alpha(1+\phi)}{\tau^h(1-\phi) + \phi\overline{p} - (1+\phi)p^h(\alpha)}\right)\frac{\partial p^h(\alpha)}{\partial \overline{p}(\alpha^s)}\frac{\partial \overline{p}(\alpha^s)}{\partial \underline{m}} \tag{189}$$

$$+ \frac{\alpha\phi}{\tau^h(1-\phi) + \phi\overline{p} - (1+\phi)p^h(\alpha)}\frac{\partial \overline{p}(\alpha^s)}{\partial \underline{m}} \tag{190}$$

$$= -\Theta^s \frac{\partial p^h(\alpha)}{\partial \overline{p}(\alpha^s)}\frac{\partial \overline{p}(\alpha^s)}{\partial \underline{m}} \tag{191}$$

$$+ \frac{\alpha\phi}{\tau^h(1-\phi) + \phi\overline{p} - (1+\phi)p^h(\alpha)}\left(1 - \frac{\partial p^h(\alpha)}{\partial \overline{p}(\alpha^s)}\right)\frac{\partial \overline{p}(\alpha^s)}{\partial \underline{m}} < 0. \tag{192}$$

□

C Data Appendix

Cash-futures basis. The cash-futures basis is the 5-year maturity (FV contract) series constructed as in Barth and Kahn (2025), defined as the implied repo rate minus an equivalent-maturity Treasury bill yield, smoothed with a 90-day centered moving average and expressed in basis points. For each day, the contract with the highest volume is used to avoid noise introduced by traders rolling into the next-to-deliver contract. We focus on the 5-year tenor because the 2- and 10-year futures series have data-quality gaps at the boundaries of our extended sample window, which runs from January 2017 through February 2025.[33]

Treasuries outstanding. Daily Treasuries outstanding are measured as the daily Total Public Debt Outstanding from the Treasury's Debt-to-the-Penny publication minus the stock of Treasury bills outstanding, obtained monthly from the Monthly Statement of the Public Debt. The outstanding stock of bills is held constant within each month. Quantities are denominated in trillions of dollars. This series is the empirical counterpart of the outstanding Treasury bond supply b_t^τ that enters the supply-dependent transaction cost in the quantitative section.

Reserves outstanding. Daily reserves outstanding are derived from two Federal Reserve Economic Data (FRED) series: Reserve Balances with Federal Reserve Banks (RESBALNSW, weekly) through early September 2020, and Total Reserves of Depository Institutions (TOTRESNS,

[33]Due to a combination of data quality concerns and the presence of Treasury security scarcity weighing on repo rates and the cash-futures basis before 2017 as indicated by high settlement fails, we begin our sample in January 2017.

monthly) thereafter, adjusted daily by subtracting cumulative daily changes in the Treasury General Account (TGA) balance obtained from the Daily Treasury Statement. Specifically, for any date t, reserves outstanding are computed as:

$$\text{Reserves}_t = \text{Reserves}_{t^\star}^{\text{FRED}} - \sum_{\tau=t^\star}^{t} \Delta\text{TGA}_\tau,$$

where $t^\star$ is the date of the most recent FRED observation. Reserves are expressed in trillions of dollars.

OLS specification for Figure 2. We estimate the following baseline specification using daily data:

$$\text{Basis}_t = \alpha + \beta_B\,\text{Treasuries}_t + \beta_M\,\text{Reserves}_t + \varepsilon_t, \tag{193}$$

where Basis_t is the cash–futures basis (in basis points), and Treasuries_t and Reserves_t represent the quantities of Treasuries (in trillions of dollars) and reserves outstanding (in trillions of dollars), respectively. Table 3 reports coefficient estimates used to generate the fitted series in Figure 2. The estimated coefficients can be interpreted as the response of the basis, in basis points, per \$1 trillion increase in the respective explanatory variables.

Repo-rate spike days (calibration of λ). We identify repo-rate spike days as days on which the spread between the DTCC GCF Repo rate for Treasury collateral (the inter-dealer general-collateral repo rate cleared through the Fixed Income Clearing Corporation (FICC)) and the interest on reserve balances (IORB; before late July 2021, the interest on excess reserves, IOER) exceeds 10 basis points. Over January 2017 – February 2025, there are 139 such days among the 2,027 trading days with available GCF repo-rate data—approximately 17 per year—concentrated around quarter-ends, tax deadlines, and large Treasury coupon issuances. The Poisson arrival rate λ is calibrated to match this count one-for-one, giving $\lambda = 139/2{,}027 = 0.069$ events per trading day.

Empirical probability of repo spikes (validation). For the untargeted validation exercise in Table 2, the empirical probability of a repo spike is computed as a centered 60-trading-day moving average of a daily indicator equal to one on days on which the GCF–IORB spread exceeds 5 basis points (days with missing GCF data enter as non-spike days). The lower 5-basis-point threshold provides a denser signal suited to tracking the temporal pattern of repo-market pressure; the rarer 10-basis-point stress events are reserved for the calibration of λ described above.

Hedge-fund repo borrowing (validation). Quarterly hedge-fund repo borrowing is obtained from SEC Form PF filings, accessed via the OFR Hedge Fund Monitor, aggregated across qualifying hedge funds over 2017–2024 ($N = 32$ quarters). The series is used only in the untargeted validation exercise in Table 2.

Deposit share (calibration of α^s). As a reference for the deposit share, we construct the share of core bank deposits in households' money-like holdings, defined as core deposits relative to the sum of core deposits and money market fund shares. Core deposits are total deposits at commercial banks minus large time deposits (FRED weekly series DPSACBW027SBOG and LTDACBW027SBOG, from the Federal Reserve's H.8 release); money market fund shares are from the Financial Accounts of the United States (series MMMFFAQ027S, quarterly), interpolated to the daily frequency of the estimation sample. This series averages roughly 0.74 over January 2017 – February 2025, and we calibrate the steady-state deposit share to this sample mean, $\alpha^s = 0.74$.

Treasuries outstanding (trillions)	4.707***
	(0.075)
Reserves outstanding (trillions)	-19.281***
	(0.288)
Constant	-39.104***
	(1.185)
R^2	0.70
Observations	2036

Table 3: This table reports estimates from the ordinary least square linear regression (193) of the cash–futures basis on Treasuries and reserves outstanding between January 2017 and February 2025. The basis is the 5-year tenor series from Barth and Kahn (2025), smoothed with a 90-day centered moving average. Treasuries outstanding equal the daily Total Public Debt Outstanding (Debt-to-the-Penny) minus the monthly stock of Treasury bills from the Monthly Statement of the Public Debt, held constant within each month. Reserves outstanding are the level from FRED's Reserve Balances with Federal Reserve Banks (RESBALNSW, weekly) through early September 2020 and Total Reserves of Depository Institutions (TOTRESNS, monthly) thereafter, adjusted by subtracting the cumulative daily change in the TGA from the Daily Treasury Statement between FRED anchor dates (i.e., the last FRED observation minus the cumulative daily change in the TGA until the next FRED anchor). Robust standard errors are in parentheses. ***, **, and * denote significance at the 1%, 5%, and 10% levels, respectively.

Internet Appendix

This Internet Appendix develops a rational-expectations extension in which the supply of reserves and Treasuries follows a Markov regime process that agents internalize, and shows that doing so leaves the model's basis predictions essentially unchanged.

IA.1 Rational-Expectations Regime Extension

Setup. In the main text the model is estimated under the assumption that the supply pair $(\underline{m}_t, b_t^\tau)$ is constant at a steady-state level, while the empirical exercise feeds in the observed time variation. Since reserves market clearing implies $m_t = \underline{m}_t$ at all dates, we drop the underline and write m_t for reserve supply throughout. This appendix instead treats (m_t, b_t^τ) as a finite-state Markov process: agents know their current regime, know the transition probabilities to every other regime, and price the risk of switching into today's portfolio choices.

We discretize Treasury supply b_t^τ into $K = 10$ regimes by K-means over the January 2017–February 2025 estimation sample. We cluster on Treasury supply because it is the dominant driver of the basis—a larger float raises the dealer intermediation cost ν—and because it rises near-monotonically, so the regimes form a *supply ladder*: we order them ascending in b^τ, and the supply state moves slowly through the ladder over time. Each regime j is summarized by its mean supply $(\bar{m}_j, \bar{b}_j^\tau)$, with $\bar{m}_j$ the within-regime average reserve level, and by the empirical daily transition matrix Q. Table IA.1 reports the classification.

Regime dynamics. Within a regime the economy behaves exactly as in the main text: from the steady state α^s, a flight-to-deposit shock (rate λ) draws an α-shock α', and a reversion shock (rate λ') returns the economy to α^s. Regime switches occur at the steady state: an independent Poisson shock (rate μ_j) moves the economy from (α^s, j) to a new steady state (α^s, j') with probability $q_{jj'}$, changing the supply $(\bar{m}_j, \bar{b}_j^\tau) \mapsto (\bar{m}_{j'}, \bar{b}_{j'}^\tau)$ but leaving α unchanged.

The friction. The extension turns on one assumption about the transaction cost ν. In the main text ν is the cost of intermediating Treasuries in the *secondary* market: it prices the fire-sale round trip, in which the shadow bank sells Treasuries to the traditional bank under an α-shock and buys them back at α^s. A regime switch is a different event—a change in the *supply* of Treasuries—which we take to be absorbed in the *primary* market, frictionlessly, in the amount by which the float changes, $\Delta b^\tau = \bar{b}_{j'}^\tau - \bar{b}_j^\tau$. We adopt this for three reasons. First, it preserves the tractability of the framework: it leaves the within-regime marginal values untouched, so each regime is solved exactly as in the main text. Second, it keeps the friction where the basis lives—in the secondary market for liquidity—rather than layering on a supply-side friction that the data give no reason to add. Third, it matches how Treasury supply is placed in practice: new issuance is taken down through regular, competitive auctions by primary dealers (Garbade and Ingber, 2005), and securities clear at auction at a small concession to buyers rather than at the secondary-market cost ν—if anything, buyers are paid to absorb it (Herb, 2025). The restriction to Δb^τ also keeps the equilibrium well-defined—an unbounded costless channel would let the shadow bank, which values Treasuries at ν above their price, buy without limit and compete the 2ν gap to zero—while any rebalancing beyond the new issuance still pays ν.

Per-regime equilibrium. Two consequences follow. First, the marginal value of a Treasury at (α^s, j) is single-valued, as a Markov equilibrium requires: the costless supply transition imposes no transaction-cost wedge, so the marginal value is pinned by the within-regime α-dynamics alone. Lemma A1 then applies regime by regime—the fire-sale round trip gives $\theta_j(\alpha^s) = -\nu_j$ and $\overline{\theta}_j(\alpha^s) = +\nu_j$, so the shadow–traditional marginal-value gap is $2\nu_j$, exactly as in a single regime. Second, the equilibrium is solved regime by regime: at the supply $(\bar{m}_j, \bar{b}^\tau_j)$, market clearing determines the bank Treasury positions and the steady-state triparty–deposit spread $\Theta^s_j \equiv r^{pt}(\alpha^s) - r^d(\alpha^s)$, with the marginal values held at the α-cycle boundaries. The spread Θ^s_j solves the steady-state condition stated next, and the basis is $-\Theta^s_j/2$.

Internalizing regime switching. Internalizing the switching alters only the bank value functions, through one extra Poisson jump; we derive the spread condition from them. The main-text value functions, now indexed by regime, stay linear in Treasury holdings,

$$V_j(b;\alpha) = \xi_j(\alpha) + \theta_j(\alpha)\, b, \qquad \overline{V}_j(\bar{b};\alpha) = \overline{\xi}_j(\alpha) + \overline{\theta}_j(\alpha)\, \bar{b},$$

so the marginal values are $V_{j,b} = \theta_j$ and $\overline{V}_{j,b} = \overline{\theta}_j$. At the steady state of regime j the shadow bank faces the flight shock at rate λ (drawing $\alpha' \sim f$) and the regime shock at rate $\mu_j = \sum_{j' \neq j} q_{jj'}$ (moving to (α^s, j') at rate $q_{jj'}$). A regime switch changes the supply of Treasuries, which the bank absorbs at no transaction cost; its continuation value is then $\overline{V}_{j'}(\bar{b};\alpha^s)$, with nothing deducted for the change in holding, and—the value function being linear in the holding—the marginal value carried into regime j' is $\overline{\theta}_{j'}(\alpha^s)$, whatever the level. The regime-j value function therefore solves

$$\rho\, \overline{V}_j(\bar{b};\alpha^s) = \overline{\pi}_j(\bar{b}) + \lambda \int \big[\overline{V}_j(\bar{b};\alpha') - \overline{V}_j(\bar{b};\alpha^s)\big] f(\alpha')\, d\alpha' + \sum_{j' \neq j} q_{jj'} \big[\overline{V}_{j'}(\bar{b};\alpha^s) - \overline{V}_j(\bar{b};\alpha^s)\big],$$

with $\overline{\pi}_j$ the flow payoff, whose derivative $\partial \overline{\pi}_j / \partial \bar{b} = r^\delta_j(\alpha^s) - r^p_j(\alpha^s)$ is the cash–futures spread. Differentiating in $\bar{b}$ (using $\overline{\theta}_j(\alpha') = -\nu_j$ in the fire-sale region $\mathcal{F}_j$), and repeating for the traditional bank (marginal flow $r^\delta_j - r^d_j - \phi\vartheta^e_j$, with $\theta_j(\alpha') = +\nu_j$ in $\mathcal{F}_j$), gives the two steady-state envelopes

$$(\rho+\lambda+\mu_j)\,\overline{\theta}_j(\alpha^s) = r^\delta_j(\alpha^s) - r^p_j(\alpha^s) + \lambda\int \overline{\theta}_j(\alpha')\,f(\alpha')\,d\alpha' + \sum_{j'\neq j} q_{jj'}\,\overline{\theta}_{j'}(\alpha^s), \tag{IA.1}$$

$$(\rho+\lambda+\mu_j)\,\theta_j(\alpha^s) = r^\delta_j(\alpha^s) - r^d_j(\alpha^s) - \phi\vartheta^e_j(\alpha^s) + \lambda\int \theta_j(\alpha')\,f(\alpha')\,d\alpha' + \sum_{j'\neq j} q_{jj'}\,\theta_{j'}(\alpha^s). \tag{IA.2}$$

The last term in each is the only new object: at a switch the carried bond is worth the destination marginal value $\overline{\theta}_{j'}(\alpha^s)$, resp. $\theta_{j'}(\alpha^s)$—continuous, with no $\pm\nu$ transaction-cost wedge, exactly because the supply change is frictionless. Subtract (IA.2) from (IA.1); using $\overline{\theta}_j(\alpha^s)-\theta_j(\alpha^s) = 2\nu_j$ (Lemma A1), $r^p_j - r^{pt}_j = \phi\vartheta^e_j$, $\Theta^s_j \equiv r^{pt}_j(\alpha^s) - r^d_j(\alpha^s)$, and $\nu_j = \nu_0(\bar{b}^\tau_j/\bar{b}^\tau)^{\gamma_\nu}$, this rearranges to

$$\Theta^s_j + 2(\rho+\lambda+\mu_j)\,\nu_j + 2\lambda\int_{\alpha'\in\mathcal{F}_j} \nu_j\,f(\alpha')\,d\alpha' \;-\; \lambda\int_{\alpha'\notin\mathcal{F}_j} \big(\overline{\theta}_j(\alpha')-\theta_j(\alpha')\big)\,f(\alpha')\,d\alpha' \;=\; \sum_{j'\neq j} q_{jj'}\,2\nu_{j'}. \tag{IA.3}$$

The right-hand side is the regime-switch continuation of the gap, $\sum_{j'\neq j} q_{jj'}\big[\overline{\theta}_{j'}(\alpha^s)-\theta_{j'}(\alpha^s)\big] = \sum_{j'\neq j} q_{jj'}\,2\nu_{j'}$, the destination gap being $2\nu_{j'}$ by Lemma A1 in regime j'. The left-hand side is the main text's constant-supply condition with the discount $\rho+\lambda$ raised to $\rho+\lambda+\mu_j$; setting $\mu_j = 0$, so both new pieces vanish, recovers it. The condition is solved exactly, one regime at a time (its right-hand side is fixed by supply). Writing $\Theta^{s,0}_j$ for the solution at $\mu_j = 0$—the constant-supply spread—the change in the basis from internalizing switching is $\delta_j \equiv \big(-\tfrac{1}{2}\Theta^s_j\big) - \big(-\tfrac{1}{2}\Theta^{s,0}_j\big) = \tfrac{1}{2}\big(\Theta^{s,0}_j - \Theta^s_j\big)$, read off the exact per-regime solutions and reported in Table IA.3. It also admits a transparent leading-order approximation: subtracting the $\mu_j = 0$ condition from (IA.3), the fire-sale integrals and the $2(\rho+\lambda)\nu_j$ term enter both and approximately cancel—their dependence on Θ^s_j is higher order—leaving $\Theta^s_j - \Theta^{s,0}_j \approx \sum_{j'\neq j} q_{jj'}2\nu_{j'} - 2\mu_j\nu_j$, so that

$$\delta_j \approx \widetilde{\delta}_j \equiv \sum_{j'\neq j} q_{jj'}\,(\nu_j - \nu_{j'}),$$

the switch-rate–weighted change in the transaction cost across regimes. This approximation $\widetilde{\delta}_j$ captures the sign and structure of the correction but, by dropping the $O(\rho+\lambda)$ discount factor, overstates the magnitude of the exact δ_j by roughly an order of magnitude. Because δ_j is the *change* in ν across a switch (not its level), weighted by the switch intensity $q_{jj'}$, and adjacent regimes differ in ν only marginally while being highly persistent ($q_{jj'}$ of order one per year), $|\delta_j| \leq 5.2\times10^{-5}$ basis points (Table IA.3)—negligible even though $\nu(b^\tau)$ sets the cross-regime basis *level* itself. The calibrated and estimated parameters are those of the main text (Table 1).

Solution algorithm.

1. *Classify.* Cluster the daily Treasury supply $\{b^\tau_t\}$ into K regimes by K-means; order them ascending in mean supply and set $(\bar{m}_j, \bar{b}^\tau_j)$ to the within-regime means.

Table IA.1: Empirical regime classification (K-means, $K = 10$, on Treasury supply b^τ, ordered ascending, 2017–2025). Reserves $\bar{m}$ and Treasury supply $\bar{\tau}^h$ are in trillions of dollars; mean duration in days.

Regime	$\bar{m}$	$\bar{\tau}^h$	Persistence	Mean duration
1	2.21	18.42	0.997	371.0
2	1.69	19.58	0.997	309.0
3	2.04	21.02	0.994	175.0
4	2.93	22.08	0.993	136.0
5	3.76	23.25	0.991	112.0
6	4.27	24.59	0.992	130.0
7	3.60	26.33	0.992	130.0
8	3.22	27.49	0.990	105.0
9	3.46	28.53	0.982	55.2
10	3.37	29.70	0.992	127.0

2. *Transition matrix.* Build Q from the observed regime sequence, restricted to adjacent transitions (tridiagonal); the exit rate is $\mu_j = \sum_{j' \neq j} q_{jj'}$.
3. *Per-regime basis.* For each j, solve (IA.3) for Θ^s_j (the right-hand side is fixed by supply, so the regimes are independent); the basis is $-\Theta^s_j/2$, with $\nu_j = \nu_0(\bar{b}^\tau_j/\bar{b}^\tau)^{\gamma_\nu}$.
4. *Map to dates.* Assign each date t to its regime $j(t)$; the model basis is $-\Theta^s_{j(t)}/2$, censored to the structural floor $\rho\,\nu_{j(t)}$ in the 2020–2021 supplementary-leverage-ratio (SLR) relief window, as in the main text.

Empirical regime classification. Table IA.1 reports each regime's mean supply $(\bar{m}_j, \bar{b}^\tau_j)$, its daily persistence, and its mean duration. The regimes run from the low-supply configuration of 2017–2018 (regime 1) to the high-supply quantitative-tightening configuration of 2023–2025 (regime 10). They are persistent—daily persistence between 0.98 and 1.00, mean durations from about two months to a year—so the supply state turns over slowly.

Transition matrix. Table IA.2 reports the estimated daily transition matrix Q, restricted to adjacent transitions (tridiagonal). Each regime stays put with probability 0.98–1.00 and otherwise moves one rung—almost always up, since Treasury supply rises near-monotonically, and occasionally down in the high-supply regimes. The supply thus drifts gradually through neighboring regimes rather than jumping, mirroring the slow rise of Treasury supply in Figure 11.

Quantitative result. Table IA.3 reports the per-regime basis $-\Theta^s_j/2$ and its change δ_j from the constant-supply value. The basis ranges from near zero to 37 basis points across the supply ladder, while δ_j is at most 5.2×10^{-5} basis points—four orders of magnitude below the ~ 0.5 basis-point precision of the 90-day moving-average data target. Internalizing rational-expectations regime switching is therefore quantitatively irrelevant, exactly as the cancellation

Table IA.2: Estimated daily transition matrix Q ($K = 10$): entry (j, j') is the probability of moving from regime j today to j' tomorrow. Q is tridiagonal: supply moves only to an adjacent regime.

Regime	1	2	3	4	5	6	7	8	9	10
1	0.997	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000
2	0.000	0.997	0.003	0.000	0.000	0.000	0.000	0.000	0.000	0.000
3	0.000	0.000	0.994	0.006	0.000	0.000	0.000	0.000	0.000	0.000
4	0.000	0.000	0.000	0.993	0.007	0.000	0.000	0.000	0.000	0.000
5	0.000	0.000	0.000	0.000	0.991	0.009	0.000	0.000	0.000	0.000
6	0.000	0.000	0.000	0.000	0.000	0.992	0.008	0.000	0.000	0.000
7	0.000	0.000	0.000	0.000	0.000	0.000	0.992	0.008	0.000	0.000
8	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.990	0.010	0.000
9	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.009	0.982	0.009
10	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.000	0.008	0.992

Table IA.3: Per-regime basis $-\Theta_j^s/2$ and its change δ_j from the constant-supply value ($\mu_j = 0$), in basis points.

Regime	Basis	δ_j
1	10.097	-7.8×10^{-6}
2	20.023	-1.3×10^{-5}
3	20.749	-1.8×10^{-5}
4	11.559	-2.8×10^{-5}
5	2.327	-4.3×10^{-5}
6	0.031	-5.2×10^{-5}
7	15.823	-3.7×10^{-5}
8	28.651	-4.4×10^{-5}
9	28.894	-8.2×10^{-6}
10	37.207	4.4×10^{-5}

in (IA.3) shows: the per-regime steady states are the equilibrium. Figure 11 plots the model basis against the observed series; the regime step tracks the level of the basis as Treasury supply rises, reaching the stress band in the high-supply 2023–2025 regime.

Discussion. The effect is negligible for two compounding reasons, both visible in (IA.3). With mean regime durations of months to a year, the exit rate μ_j is small, so agents heavily discount future switches; and the two terms that switching adds—a faster discount and a destination continuation—nearly cancel, because the marginal-value gap $2\nu_j$ moves little across adjacent supply regimes. The conclusion is robust to the number of regimes K: it rests only on the slow, near-monotone drift of Treasury supply, which any reasonable discretization preserves.

Replication notes. The figure and tables are reproduced from `code/matlab/re_regimes_v7/` in the replication package:

```
matlab -batch "make_regimes"    % Tables IA.1-IA.3 + Figure regimes.pdf
```

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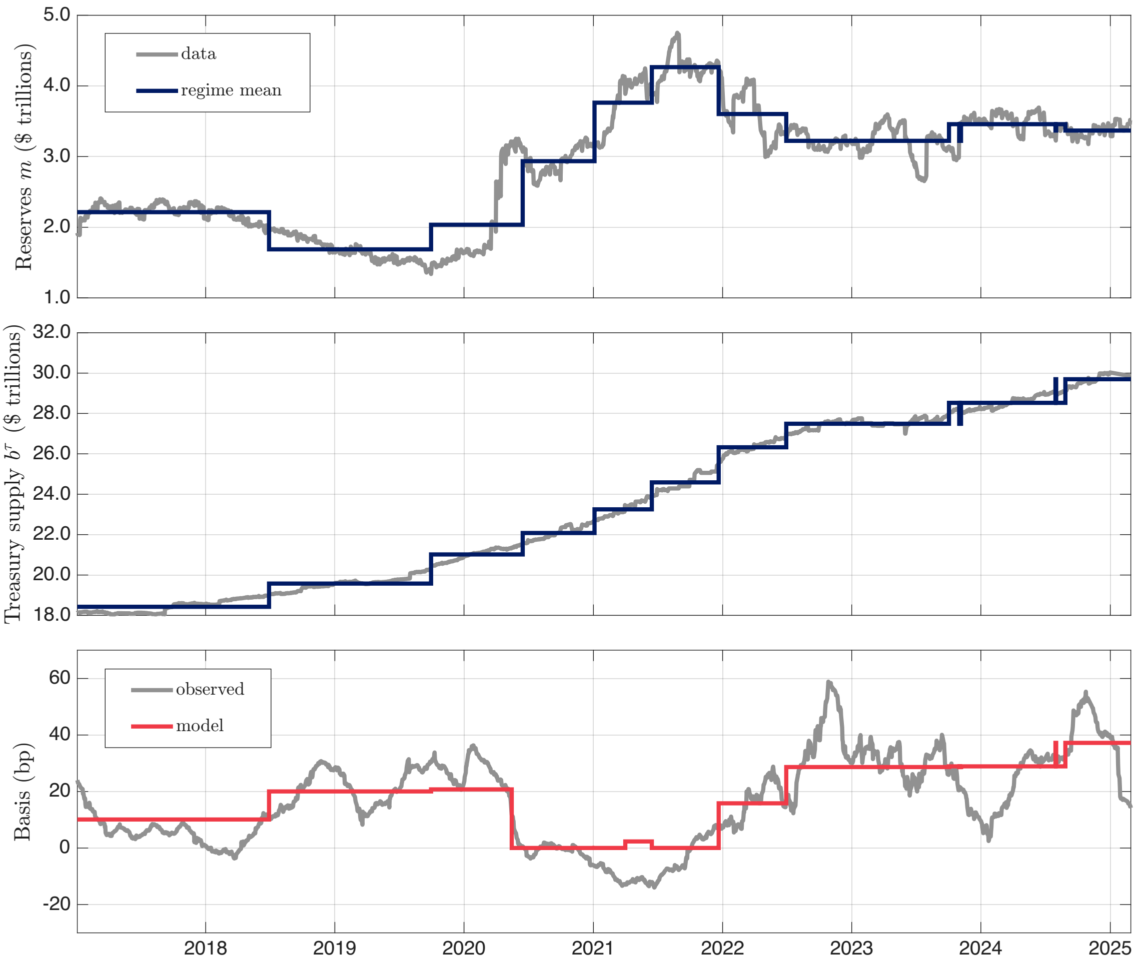

Figure 11: The supply-regime classification and the model basis. *Top:* reserves m_t; *middle:* Treasury supply b_t^{τ}; each with the regime-mean step (blue) over the daily series (grey), showing which of the ten regimes each date is assigned to. *Bottom:* the observed cash–futures basis (grey) and the per-regime model basis (red). The model step tracks the level of the basis as Treasury supply rises, reaching the stress band in the high-supply 2023–2025 regime; it stays below the short-lived moving-average peaks, which lie above the model's steady-state basis at any observed supply. In the 2020–2021 SLR-relief window the model basis is censored to its structural floor, as in the main-text estimation.